Undergraduate Research via Experimental Mathematics

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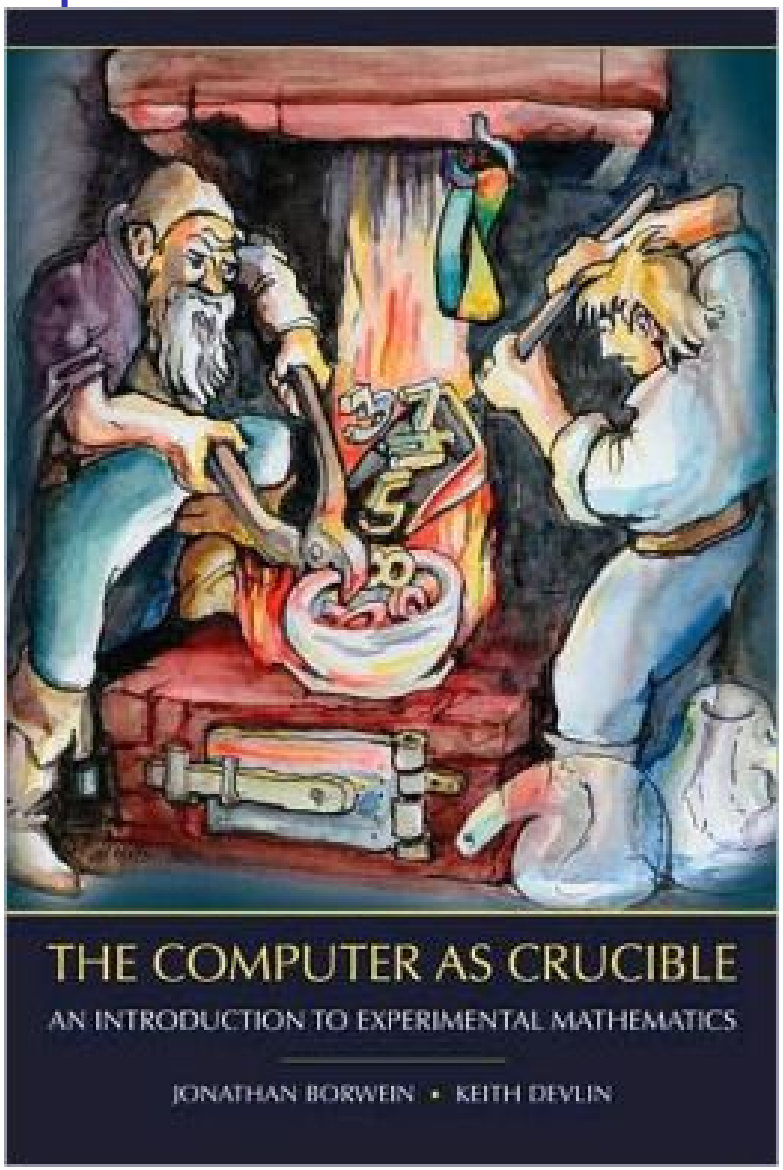
2025 Joint Mathematics Meetings
AMS Special Session on Bringing Research into the Math Classroom
January 5, 2026

Experimental Math is...



"Experimental math is the use of a computer to **run computations** to **look for patterns**, to **identify** particular **numbers and sequences**, to **gather evidence** in support of specific mathematical assertions that may themselves arise by computational means."

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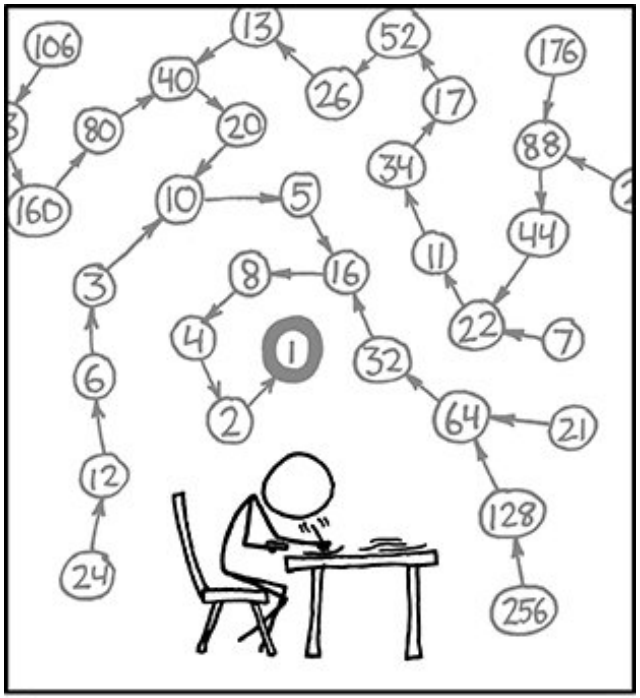
- Spring 2015, 2017, 2019: Experimental Math runs as a regular elective
- Since 2020: fewer elective courses because of lower enrollment

  Caveat: This talk describes a past course.

Experimental Math (Valpo style...)

- **Throughout the course:**
  - Mini-essays on philosophy of doing math
  - Individualized project
- **Intro: (1.5 weeks)**
  - What *is* experimental math?
  - Making friends with the computer
- **Guided exploration: (11.5 weeks)**
  - Introduce a new problem
  - Program together
  - List of "experiments" in groups
- **Wrap up (2 weeks):**
  - Landmarks of computers in proofs
    (Four color theorem, Kepler conjecture)
  - Student showcase

Lesson 1: Collatz Conjecture



THE COLLATZ CONJECTURE STATES THAT IF YOU PICK A NUMBER, AND IF IT'S EVEN DIVIDE IT BY TWO AND IF IT'S ODD MULTIPLY IT BY THREE AND ADD ONE, AND YOU REPEAT THIS PROCEDURE LONG ENOUGH, EVENTUALLY YOUR FRIENDS WILL STOP CALLING TO SEE IF YOU WANT TO HANG OUT.

Source: `http://xkcd.com/710/`

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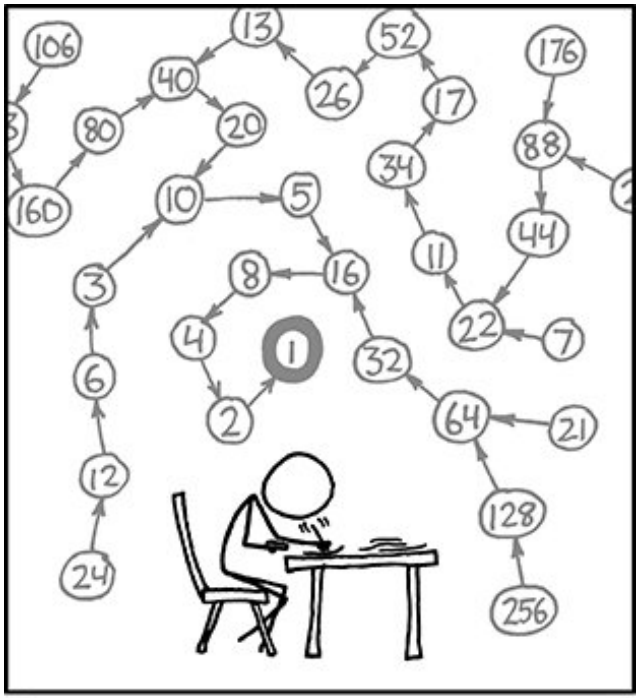

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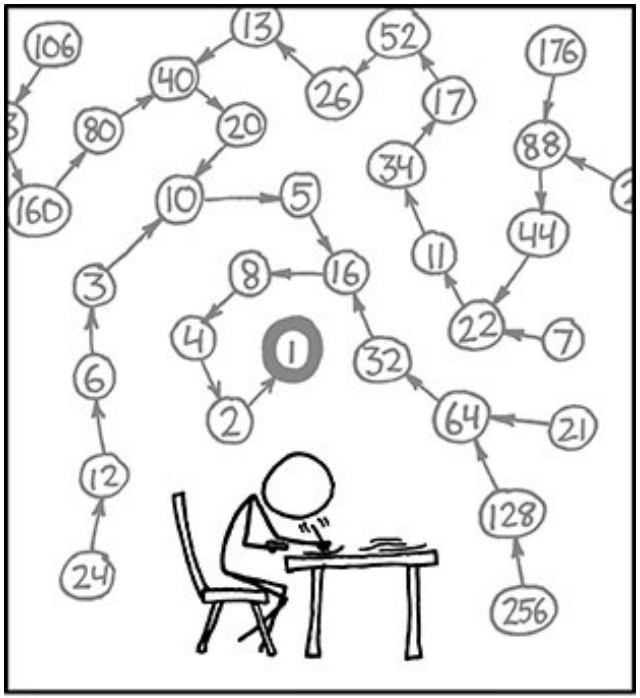

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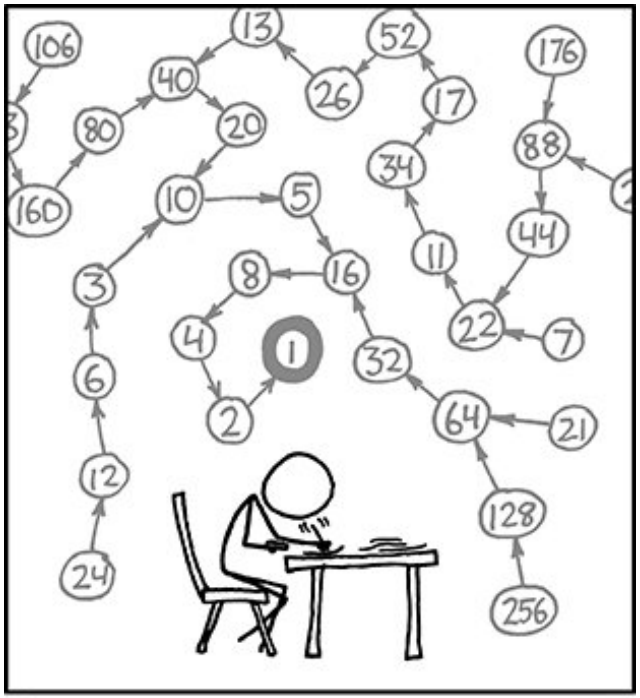

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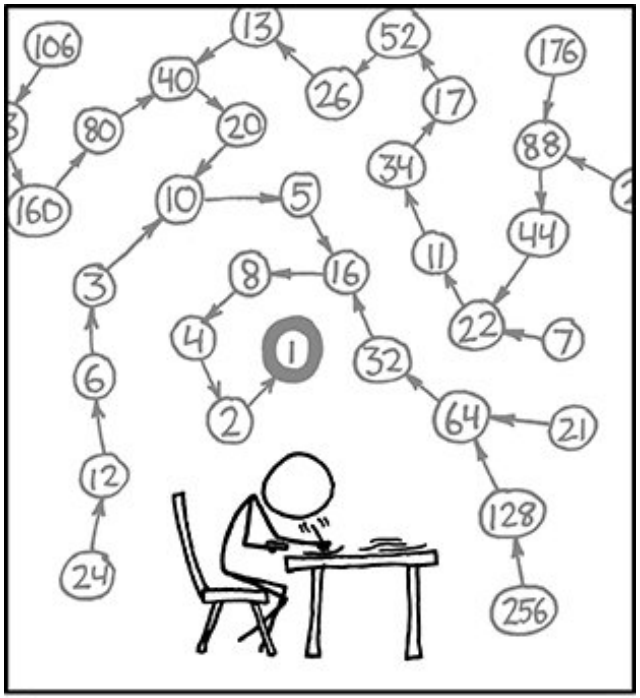

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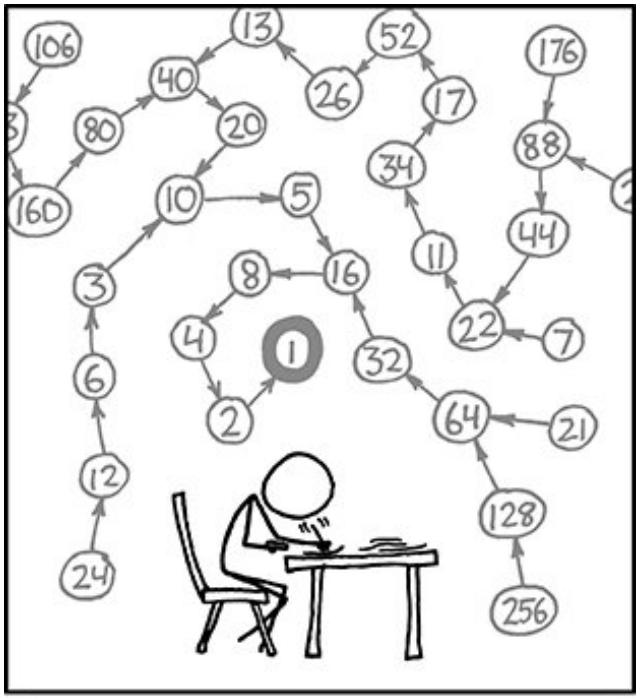

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- **Bonus: Try your own piecewise functions.**

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- Calculating π
- What is the quadrillionth decimal place of π?
- What is that number?
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- Evaluate the following integral

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Other topics...

- Combinatorial sequences
- Continued fractions and irrationality
- Graph coloring

Transitions

 = in-class.

 = student-generated experiment.

	problem statement	code	data
early classes			
later classes			
final project			

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(15 week semester)

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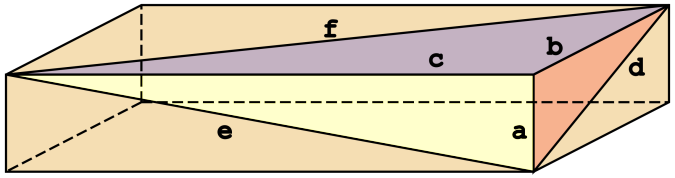
  (clarity, difficulty, correctness)

Project examples

(Spring 2019)

- Euler bricks
- Sports analogs of the ballot problem
- Modifying the Kolakoski Sequence



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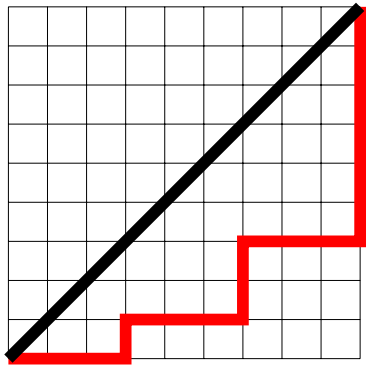

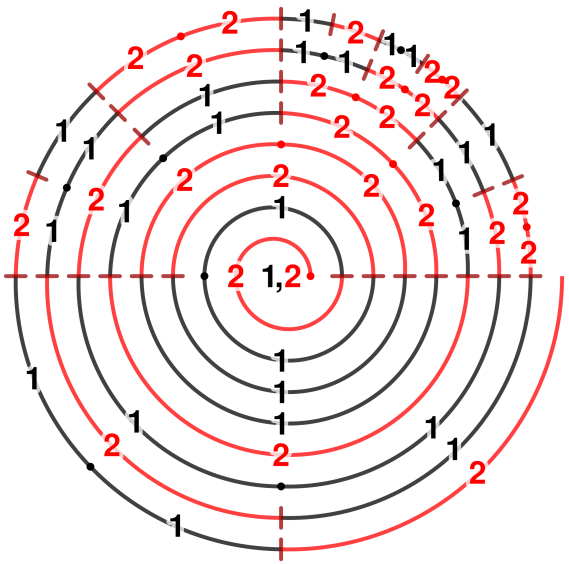

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From an engineering major:

I was always taught: here is a concept, here is what it does, here is how to do it. I figured stuff that I need to learn would always just be given to me. This class has given me an appreciation for actually getting to explore concepts and learn on my own, which is something I would previously never thought would have worked.

From past to present...

- How would course structure change if primary goal was research?

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- How would course structure change if primary goal was research?
- How would course content change in era of AI-assisted coding?

Thanks for listening!

Email: Lara.Pudwell@valpo.edu
Slides at: faculty.valpo.edu/lpudwell