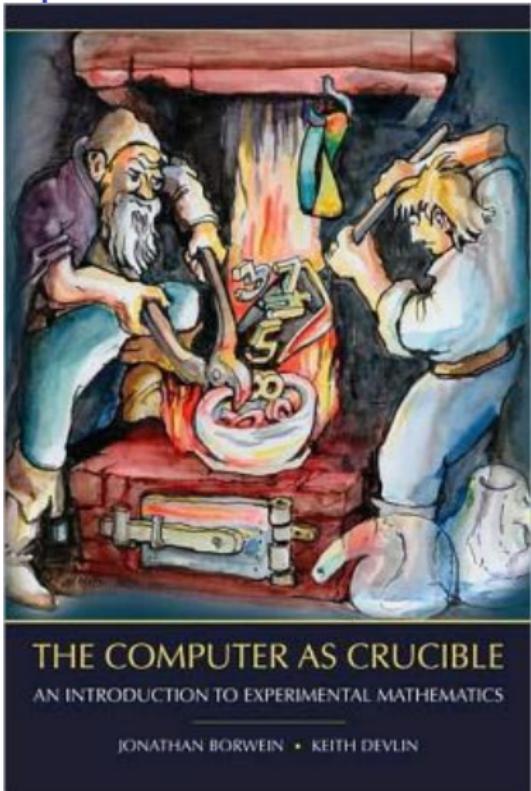


# Undergraduate Research via Experimental Mathematics

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2025 Joint Mathematics Meetings  
AMS Special Session on Bringing Research into the Math Classroom  
January 5, 2026

# Experimental Math is...



“Experimental math is the use of a computer to **run computations** to **look for patterns**, to **identify** particular **numbers and sequences**, to **gather evidence** in support of specific mathematical assertions that may themselves arise by computational means.”

# Experimental Math at Valpo

- Spring 2009: need a special topics course

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- Spring 2015, 2017, 2019: Experimental Math runs as a regular elective
- Since 2020: fewer elective courses because of lower enrollment

Caveat: This talk describes a past course.

# Experimental Math (Valpo style...)

- **Throughout the course:**

- ▶ Mini-essays on philosophy of doing math
- ▶ Individualized project

- **Intro: (1.5 weeks)**

- ▶ What *is* experimental math?
- ▶ Making friends with the computer

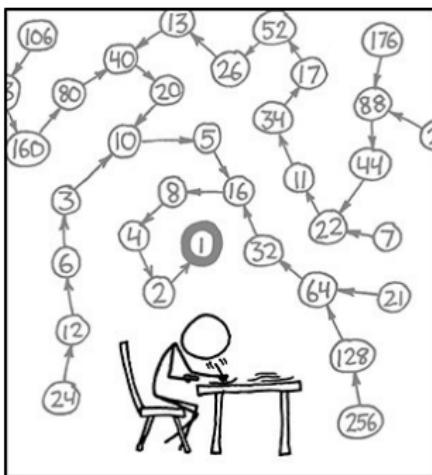
- **Guided exploration: (11.5 weeks)**

- ▶ Introduce a new problem
- ▶ Program together
- ▶ List of “experiments” in groups

- **Wrap up (2 weeks):**

- ▶ Landmarks of computers in proofs  
(Four color theorem, Kepler conjecture)
- ▶ Student showcase

# Lesson 1: Collatz Conjecture



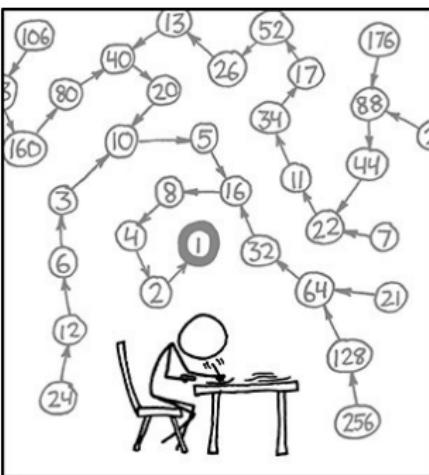
THE COLLATZ CONJECTURE STATES THAT IF YOU PICK A NUMBER, AND IF IT'S EVEN DIVIDE IT BY TWO AND IF IT'S ODD MULTIPLY IT BY THREE AND ADD ONE, AND YOU REPEAT THIS PROCEDURE LONG ENOUGH, EVENTUALLY YOUR FRIENDS WILL STOP CALLING TO SEE IF YOU WANT TO HANG OUT.

Source: <http://xkcd.com/710/>

# Lesson 1: Collatz Conjecture

In class...

- $f(n) = \begin{cases} 3n + 1 & n \text{ odd} \\ n/2 & n \text{ even} \end{cases}$
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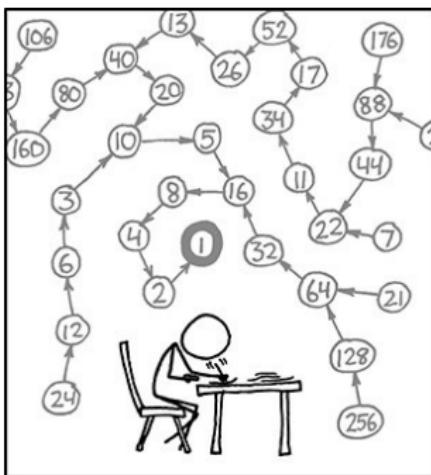


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## Experiments...

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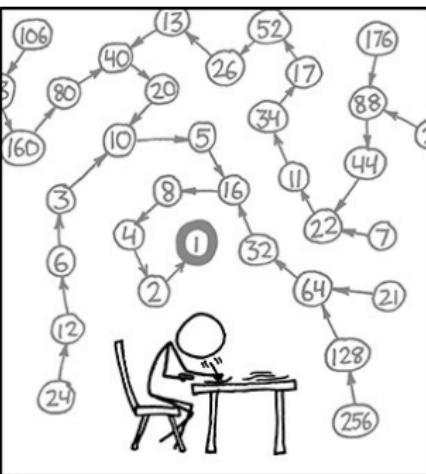
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$$f(n) = \begin{cases} 3n + 1 & n \text{ prime} \\ n/p_1 & n = p_1^{e_1} \cdots p_k^{e_k}, p_i < p_{i+1} \forall i \end{cases}$$

&lt;img alt="A xkcd comic strip. A stick figure sits at a desk, looking at a computer screen. The screen shows a complex graph of numbers connected by arrows, representing the Collatz conjecture. The numbers 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16384, 32768, 65536, 131072, 262144, 524288, 1048576, 2097152, 4194304, 8388608, 16777216, 33554432, 67108864, 134217728, 268435456, 536870912, 1073741824, 2147483648, 4294967296, 8589934592, 17179869184, 34359738368, 68719476736, 137438953472, 274877906944, 549755813888, 1099511627776, 2199023255552, 4398046511104, 8796093022208, 17592186044416, 35184372088832, 70368744177664, 140737488355328, 281474976710656, 562949953421312, 112589990684264, 225179981368528, 450359962737056, 900719925474112, 1801439850948224, 3602879701896448, 7205759403792896, 14411518807585792, 28823037615171584, 57646075230343168, 115292150460686336, 230584300921372672, 461168601842745344, 922337203685490688, 1844674407370981376, 3689348814741962752, 7378697629483925504, 14757395258967851008, 29514790517935702016, 59029581035871404032, 118059162071742808064, 236118324143485616128, 472236648286971232256, 944473296573942464512, 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# Lesson 1: Collatz Conjecture

In class...

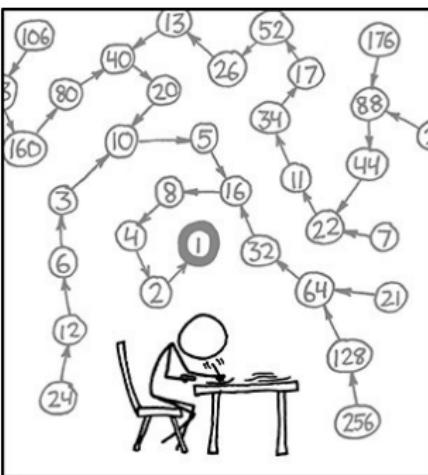
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- **Bonus: Try your own piecewise functions.**



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# Experimenting with $5n + 1$

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$9 \rightarrow 46 \rightarrow 23 \rightarrow 116 \rightarrow 58 \rightarrow 29 \rightarrow 146 \rightarrow 73 \rightarrow 366 \rightarrow 183 \rightarrow 916 \rightarrow 458 \rightarrow 229 \rightarrow 1146 \rightarrow 573 \rightarrow 2866 \rightarrow 1433 \rightarrow 7166 \rightarrow 3583 \rightarrow 17916 \rightarrow 8958 \rightarrow \dots$

# Content

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Other topics...

- Combinatorial sequences
- Continued fractions and irrationality
- Graph coloring

# Transitions



= in-class.



= student-generated experiment.

	problem statement	code	data
early classes			
later classes			
final project			

# Transitions



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	problem statement	code	data
early classes	✓	✓	💡
later classes	✓	💡	💡
final project			

# Transitions



= in-class.



= student-generated experiment.

	problem statement	code	data
early classes			
later classes			
final project			

# Scaffolding the final project

(15 week semester)

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(advice on how to pick a topic)

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(clarity, progress since last checkpoint)
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# Scaffolding the final project

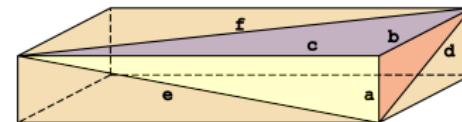
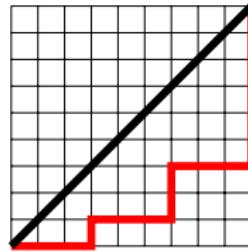
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(clarity, difficulty, correctness)

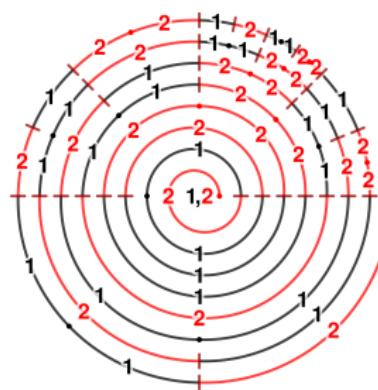
# Project examples

(Spring 2019)

- Euler bricks
- Sports analogs of the ballot problem
- Modifying the Kolakoski Sequence



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# Reaction

From a math major:

*I'm learning math isn't just about memorizing formulas and plugging in numbers, but building on what you know, asking your own questions, and realizing not everything has a known answer just yet.*

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From an engineering major:

*I was always taught: here is a concept, here is what it does, here is how to do it. I figured stuff that I need to learn would always just be given to me. This class has given me an appreciation for actually getting to explore concepts and learn on my own, which is something I would previously never thought would have worked.*

## From past to present...

- How would course structure change if primary goal was research?

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- How would course structure change if primary goal was research?
- How would course content change in era of AI-assisted coding?

# Thanks for listening!

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Slides at: [faculty.valpo.edu/lpudwell](http://faculty.valpo.edu/lpudwell)