Fostering Curiosity & Self-Directed Learning via Scaffolding

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Creating a classroom environment
that empowers students to learn independently
Green '23 Project NExT presentation
August 7, 2024

A Foundation for Self-Directed Learning

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- Agency
- Skills

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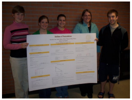

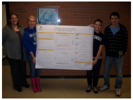
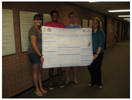
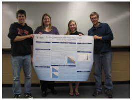
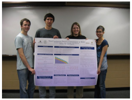
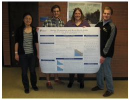


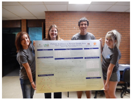
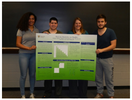
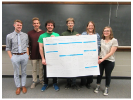
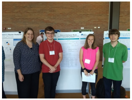
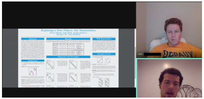

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1. What topic is interesting to *you*?
2. What questions do *you* have?

How to Read a Book by Adler and Van Doren

How to Read a Book by Adler and Van Doren

"**Mathematics is a language**, and... we can learn it like any other, including our own... **it has its own vocabulary, grammar, and syntax, and these have to be learned by the beginning reader.** Certain symbols and relationships between symbols have to be memorized. The problem is different, because the language is different, but it is no more difficult, theoretically, than learning to read English or French or German."

How to Think Like a Mathematician by Houston

How to Think Like a Mathematician by Houston

- Read with a purpose.
- Read actively. Have pen and paper with you.
- You do not have to read in sequence but read systematically.
- Ask questions.
- Read the definitions, theorems and examples first. The proofs can come later.
- Check the text by applying formulas, etc.
- Do exercises and problems.
- Move on if you are stuck.
- Write a summary.
- Reflect – What have you learned?

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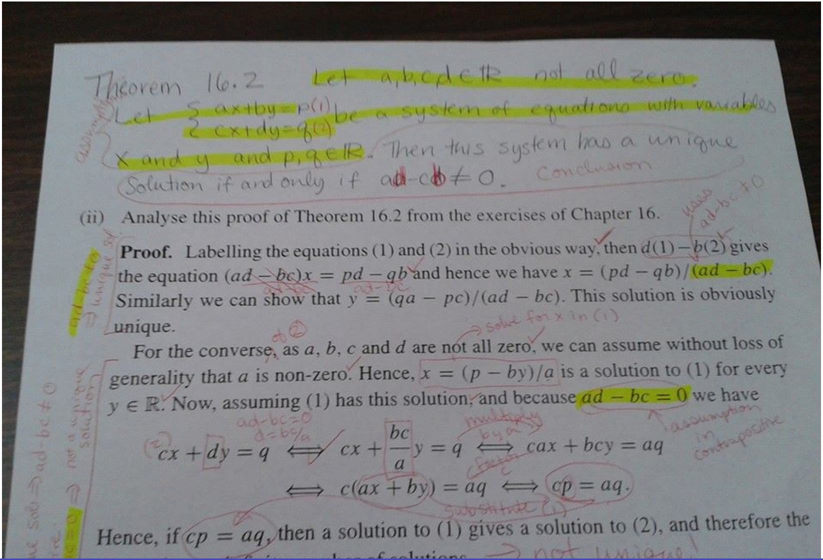
Example of Scaffolded Reading

You will be given two proofs. At your table, discuss the following:

1. What is the purpose of each paragraph?
2. What are the assumptions of the theorem? Where are they used?
3. Draw a picture if possible.
4. Apply the theorem to examples. (Are there extreme examples? Trivial examples?)
5. Apply the theorem to non-examples.
6. Check the text line by line; do you spot any mistakes?

Applying Houston's Strategy



Theorem 16.2 Let $a, b, c, d \in \mathbb{R}$ not all zero. Let $\begin{cases} ax+by=p \ (1) \\ cx+dy=q \ (2) \end{cases}$ be a system of equations with variables x and y and $p, q \in \mathbb{R}$. Then this system has a unique solution if and only if $ad-cb \neq 0$.

(ii) Analyse this proof of Theorem 16.2 from the exercises of Chapter 16.

Proof. Labelling the equations (1) and (2) in the obvious way, then $d(1)-b(2)$ gives the equation $(ad-bc)x = pd-qb$ and hence we have $x = (pd-qb)/(ad-bc)$. Similarly we can show that $y = (qa-pc)/(ad-bc)$. This solution is obviously unique.

For the converse, as a, b, c and d are not all zero, we can assume without loss of generality that a is non-zero. Hence, $x = (p-by)/a$ is a solution to (1) for every $y \in \mathbb{R}$. Now, assuming (1) has this solution, and because $ad-bc=0$ we have

$$cx + dy = q \iff cx + \frac{bc}{a}y = q \iff cax + bcy = aq$$
$$\iff c(ax+by) = aq \iff cp = aq.$$

Hence, if $cp = aq$, then a solution to (1) gives a solution to (2), and therefore the

Proof 1: linear algebra

Theorem 1. *Let $a, b, c, d \in \mathbb{R}$ not all zero. Let*

$$\begin{cases} ax + by = p \\ cx + dy = q \end{cases}$$

be a system of equations with variables x and y and $p, q \in \mathbb{R}$. Then this system has a unique solution if and only if $ad - bc \neq 0$.

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$$\iff c(ax + by) = aq \iff cp = aq$$

Hence, if $cp = aq$, then a solution to (1) gives a solution to (2), and therefore the system has an infinite number of solutions.

On the other hand, if $cp \neq aq$, then a solution to (1) is not a solution to (2), and therefore no solutions exist. □

Proof 2: sets

Theorem 2. *Let A and B be sets. Then we have $A \neq B$ if and only if $(A \setminus B) \cup (B \setminus A) \neq \emptyset$.*

Proof. Suppose that $A = B$. Then

$$(A \setminus B) \cup (B \setminus A) = (A \setminus A) \cup (A \setminus A) = \emptyset \cup \emptyset = \emptyset.$$

If $A \neq B$, then there exists $x \in A$ but $x \neq B$ or there exists $x \in B$ but $x \notin A$. In the former we have $x \in A \setminus B$, hence $x \in (A \setminus B) \cup (B \setminus A)$. Therefore $(A \setminus B) \cup (B \setminus A) \neq \emptyset$.

A similar reasoning proves the result in the latter possibility.

□

To Ponder:

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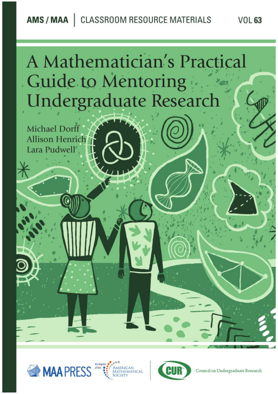
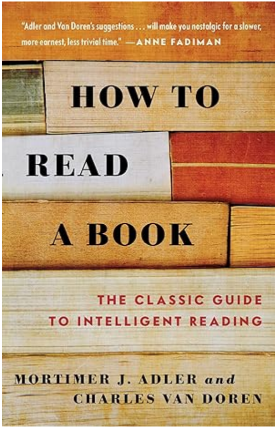
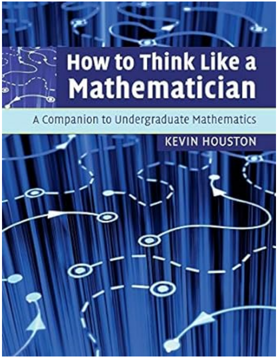
- When is reading a primary goal for your students?
- What questions would help scaffold your students towards making sense of a complicated idea?
- When do your course goals leave an opportunity for more agency?

Resources







slides at faculty.valpo.edu/lpudwell