FLsummary.pdf – output of FINLABEL for various pattern sets

GENERAL FORMAT

each entry has:
- generating function
- sequence to 30 terms
- results of searching for sequence in the online encyclopedia of integer sequences
- recurrence formula from Findrec
- asymptotic expansion from Asy
- asymptotic results of Zinn
- number of symmetry classes with this sequence/generating function
- one representative from each symmetry class

to see an expanded version with every member of every symmetry class listed, see FLoutput.pdf

SUMMARY

results for [3] pattern sets
6 sets / 2 symmetry classes / 1 Wilf class
0/2 (0 %) symmetry classes can be counted by FINLABEL
2/2 (100 %) symmetry classes can be counted by WILF.
that's 100 % in all.

results for [3,3] pattern sets
15 sets / 5 symmetry classes / 3 Wilf classes
5/5 (100 %) symmetry classes can be counted by FINLABEL.
that's 100 % in all.

results for [3,3,3] pattern sets
20 sets / 5 symmetry classes / 3 Wilf classes
5/5 (100 %) symmetry classes can be counted by FINLABEL.
that's 100 % in all.

results for [3,3,3,3] pattern sets
15 sets / 5 symmetry classes / 3 Wilf classes
5/5 (100 %) symmetry classes can be counted by FINLABEL.
that's 100 % in all.

results for [4,3] pattern sets
144 sets / 30 symmetry classes / (at least) 7 Wilf classes
12/30 (40 %) symmetry classes can be counted by FINLABEL.
10/30 (33 %) symmetry classes can be counted by WILF.
that's 73 % in all.

results for [4,4] pattern sets
9/56 (16 %) symmetry classes can be counted by FINLABEL.
that's 16 % in all.

results for [4,3,3] pattern sets
360 sets / 66 symmetry classes / 10 Wilf classes
66/66 (100 %) symmetry classes can be counted by FINLABEL.
that's 100 % in all.

results for [4,4,3] pattern sets
1656 sets / 268 symmetry classes / (at least) 40 Wilf classes
179/268 (67 %) symmetry classes can be counted by FINLABEL.
that's 67 % in all.

summarizing results for [4,4,4] pattern sets
2024 sets / 317 symmetry classes / (at least) 108 Wilf classes
116/317 (37 %) symmetry classes can be counted by FINLABEL.
that's 37 % in all.

summarizing results for [5,3] pattern sets
720 sets / 118 symmetry classes / (at least) 10 Wilf classes
15/118 (13 %) symmetry classes can be counted by FINLABEL.
that's 13 % in all.

List: [3]

6 sets
2 symmetry classes
1 Wilf class

[3]-sets, arranged by common generating function

GENERATING FUNCTION: (n/a)
sequence to 30 terms: 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845, 35357670, 129644790, 477638700, 1767263190, 6564120420, 24466267020, 91482563640, 343059613650, 1289904147324, 4861946401452, 18367353072152, 69533550916004, 263747951750360, 1002242216651368, 3814986502092304
(A000108: Catalan numbers: $C(n) = binomial(2n, n)/(n + 1) = (2n)!/(n!(n + 1)!)$)
RECURRENCE: $-\frac{2(1+2n)}{2+n} + N$
ASYMPTOTIC EXPANSION: $\frac{4^n(1-\frac{9}{8n}+145128n^2)}{n^{3/2}}$
ZINN: $a(n)$ asymptotic to $n^{-1.499011783}3.996563696^n$
THERE ARE 2 SYMMETRY CLASSES WITH THIS SEQUENCE:
{[1, 2, 3]}
{[1, 3, 2]}

summarizing results for [3] pattern sets
there are 2 symmetry classes in all.
0/2 (0 %) can be counted by FINLABEL.
2/2 (100 %) can be counted by WILF.
thus 100 % of the symmetry classes can be counted by either FINLABEL or WILF.

List: [3,3]

15 sets
5 symmetry classes
3 Wilf classes

[3,3]-sets, arranged by common generating function

GENERATING FUNCTION: $x(1 + 2x + 4x^3 + 4x^2)$
sequence to 30 terms: 1, 2, 4, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0

(not in online encyclopedia of integer sequences)
RECURRENCE: (no result)
THERE IS 1 SYMMETRY CLASS WITH THIS SEQUENCE:
{[1, 2, 3], [3, 2, 1]}

GENERATING FUNCTION: $-\frac{x(x^2-x+1)}{(x-1)^3}$
sequence to 30 terms: 1, 2, 4, 7, 11, 16, 22, 29, 37, 46, 56, 67, 79, 92, 106, 121, 137, 154, 172, 191, 211, 232, 254, 277, 301, 326, 352, 379, 407, 436
(A000124: Central polygonal numbers)
RECURRENCE: $-\frac{2+n+n^2}{2-n+n^2} + N$
ASYMPTOTIC EXPANSION: $n^2(1 - \frac{1}{n} + \frac{2}{n^2})$
ZINN: $a(n)$ asymptotic to $n^{2.040542696}1.002773049^n$
THERE IS 1 SYMMETRY CLASS WITH THIS SEQUENCE:
{[1, 2, 3], [2, 3, 1]}

GENERATING FUNCTION: $-\frac{x}{2x-1}$
sequence to 30 terms: 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16384, 32768, 65536, 131072, 262144, 524288, 1048576, 2097152, 4194304, 8388608, 16777216, 33554432, 67108864, 134217728, 268435456, 536870912
(A000079: Powers of 2: $a(n) = 2^n$.)
RECURRENCE: $-2 + N$
ASYMPTOTIC EXPANSION: (no result)
ZINN: (no result)
THERE ARE 3 SYMMETRY CLASSES WITH THIS SEQUENCE:
{[1, 2, 3], [1, 3, 2]}
{[1, 3, 2], [2, 1, 3]}
{[1, 3, 2], [2, 3, 1]}

summarizing results for [3,3] pattern sets
there are 5 symmetry classes in all.
5/5 (100 %) can be counted by FINLABEL.
thus 100 % of the symmetry classes can be counted by either FINLABEL or WILF.

List: [3,3,3]

20 sets
5 symmetry classes
3 Wilf classes

[3,3,3]-sets, arranged by common generating function

GENERATING FUNCTION: $x(1+2x+3x^2+x^3)$
sequence to 30 terms: 1, 2, 3, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
(not in online encyclopedia of integer sequences)
RECURRENCE: N^4
ASYMPTOTIC EXPANSION: (no unique dominant root)
ZINN: (no result)
THERE IS 1 SYMMETRY CLASS WITH THIS SEQUENCE:
{[1, 2, 3], [3, 1, 2], [3, 2, 1]}

GENERATING FUNCTION: $\frac{x}{(x-1)^2}$
sequence to 30 terms: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30
(A000027: The natural numbers)
RECURRENCE: $-\frac{-1+n}{n}+N$
ASYMPTOTIC EXPANSION: (no result)
ZINN: $a(n)$ asymptotic to $n^{0.9996908788}1.000598686^n$
THERE ARE 3 SYMMETRY CLASSES WITH THIS SEQUENCE:
{[1, 3, 2], [2, 1, 3], [3, 2, 1]}
{[2, 1, 3], [2, 3, 1], [3, 2, 1]}
{[1, 3, 2], [2, 1, 3], [2, 3, 1]}

GENERATING FUNCTION: $-\frac{x(1+x)}{x^2+x-1}$
sequence to 30 terms: 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269
(A000045: Fibonacci numbers: $F(n)=F(n-1)+F(n-2), F(0)=0, F(1)=1, F(2)=1, ...$)
RECURRENCE: $-1-N+N^2$
ASYMPTOTIC EXPANSION: (no result)
ZINN: $a(n)$ asymptotic to $n^{-6.643070440e-7}1.618034360^n$
THERE IS 1 SYMMETRY CLASS WITH THIS SEQUENCE:
{[2, 3, 1], [3, 1, 2], [3, 2, 1]}

summarizing results for [3,3,3] pattern sets
there are 5 symmetry classes in all.
5/5 (100 %) can be counted by FINLABEL.
thus 100 % of the symmetry classes can be counted by either FINLABEL or WILF.

List: [3,3,3,3]

15 sets
5 symmetry classes
3 Wilf classes

[3,3,3,3]-sets, arranged by common generating function

GENERATING FUNCTION: $x(1+2x+2x^2)$
sequence to 30 terms: 1, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
(not in online encyclopedia of integer sequences)
RECURRENCE: (no result)
textscthere is 1 symmetry class with this sequence are.
{[1, 2, 3], [2, 1, 3], [3, 1, 2], [3, 2, 1]}

GENERATING FUNCTION: $x(1+x)(x^2+x+1)$
sequence to 30 terms: 1, 2, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
(not in online encyclopedia of integer sequences)
RECURRENCE: (no result)
textscthere is 1 symmetry class with this sequence:
{[1, 2, 3], [2, 3, 1], [3, 1, 2], [3, 2, 1]}

GENERATING FUNCTION: $-\frac{x(1+x)}{x-1}$
sequence to 30 terms: 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2
(not in online encyclopedia of integer sequences)
RECURRENCE: (no result)
textscthere are 3 symmetry classes with this sequence:
{[1, 3, 2], [2, 1, 3], [3, 1, 2], [3, 2, 1]}
{[1, 3, 2], [2, 1, 3], [2, 3, 1], [3, 1, 2]}
{[1, 3, 2], [2, 3, 1], [3, 1, 2], [3, 2, 1]}

summarizing results for [3,3,3,3] pattern sets
there are 5 symmetry classes in all.
5/5 (100 %) can be counted by FINLABEL.
thus 100 % of the symmetry classes can be counted by either FINLABEL or WILF..

List: [4]

24 sets
7 symmetry classes
3 Wilf classes

[4]-sets, arranged by common generating function

GENERATING FUNCTION: (n/a)
sequence to 30 terms: 1, 2, 6, 23, 103, 513, 2761, 15767, 94359, 586590, 3763290, 24792705, 167078577, 1148208090, 8026793118, 56963722223, 409687815151, 2981863943718, 21937062144834, 162958355218089, 1221225517285209, 9225729232653663, 70209849031116183, 537935616492552297, 4147342550996290153, 32159907636432567578, 250717538500344886206, 1964347085978431234383, 15462159345628498316319, 122238900487877503161969
(A005802: Number of permutations in S_n with longest increasing subsequence of length ¡= 3 (i.e. 1234-avoiding permutations); vexillary permutations (i.e. 2143-avoiding).)

RECURRENCE: $\frac{9(n+1)^2}{(n+4)^2} - \frac{(41+42n+10n^2)N}{(n+4)^2} + N^2$

ASYMPTOTIC EXPANSION: $\frac{9^n(1-\frac{11}{2n}+\frac{20}{n^2})}{n^4}$

ZINN: $a(n)$ asymptotic to $n^{-3.990767318}8.979528508^n$

THERE ARE 2 SYMMETRY CLASSES WITH THIS SEQUENCE:

{[1, 2, 3, 4]}

{[2, 1, 3, 4]}

SYMMETRY CLASSES UNCOUNTED FINLABEL AND WILF (THERE ARE 5):

{[2, 4, 1, 3]}

{[2, 1, 4, 3]}

{[1, 3, 2, 4]}

{[1, 4, 2, 3]}

{[1, 4, 3, 2]}

summarizing results for [4] pattern sets

there are 7 symmetry classes in all.

0/7 (0 %) can be counted by FINLABEL.

2/7 (29 %) can be counted by WILF.

thus 29 % of the symmetry classes can be counted by either FINLABEL or WILF.

List: [4,3]

144 sets
30 symmetry classes
(at least) 8 Wilf classes

[4,3]-sets, arranged by common generating function

GENERATING FUNCTION: $\frac{x(2x^3-5x^2+3x-1)}{(x-1)^5}$
sequence to 30 terms: 1, 2, 5, 13, 30, 61, 112, 190, 303, 460, 671, 947, 1300, 1743, 2290, 2956, 3757, 4710, 5833, 7145, 8666, 10417, 12420, 14698, 17275, 20176, 23427, 27055, 31088, 35555
(not in online encyclopedia of integer sequences)
RECURRENCE: $-\frac{1}{2}(n+1)(n^3+5n^2-6n+24)+\frac{1}{2}n(34-13n+2n^2+n^3)N$
ASYMPTOTIC EXPANSION: $n^4(1+\frac{2}{n}-\frac{13}{n^2})$
ZINN: $a(n)$ asymptotic to $n^{3.939051976}1.002997833^n$
THERE IS 1 SYMMETRY CLASS WITH THIS SEQUENCE:
{[1, 2, 3], [4, 3, 1, 2]}

GENERATING FUNCTION: $-\frac{x(3x^2-3x+1)}{(x-1)(2x-1)^2}$
sequence to 30 terms: 1, 2, 5, 13, 33, 81, 193, 449, 1025, 2305, 5121, 11265, 24577, 53249, 114689, 245761, 524289, 1114113, 2359297, 4980737, 10485761, 22020097, 46137345, 96468993, 201326593, 419430401, 872415233, 1811939329, 3758096385, 7784628225
(not in online encyclopedia of integer sequences)
RECURRENCE: $4+2n+(-5-3n)N+(n+1)N^2$
ASYMPTOTIC EXPANSION: (no result)
ZINN: $a(n)$ asymptotic to $n^{0.9996728378}2.001284628^n$
THERE IS 1 SYMMETRY CLASS WITH THIS SEQUENCE:
{[1, 3, 2], [4, 3, 1, 2]}

GENERATING FUNCTION: $-\frac{x(x-1)}{x^2-3x+1}$
sequence to 30 terms: 1, 2, 5, 13, 34, 89, 233, 610, 1597, 4181, 10946, 28657, 75025, 196418, 514229, 1346269, 3524578, 9227465, 24157817, 63245986, 165580141, 433494437, 1134903170, 2971215073, 7778742049, 20365011074, 53316291173, 139583862445, 365435296162, 956722026041
(A099496: $(-1)^n Fib(2n+1)$
RECURRENCE: $1-3N+N^2$
ASYMPTOTIC EXPANSION: (no result)
ZINN: (no result)
THERE ARE 6 SYMMETRY CLASSES WITH THIS SEQUENCE:
{[2, 3, 1], [3, 2, 1, 4]}
{[1, 3, 2], [3, 2, 4, 1]}
{[1, 2, 3, 4], [1, 3, 2]}
{[2, 1, 3, 4], [1, 3, 2]}
{[3, 1, 2], [3, 2, 4, 1]}
{[4, 1, 2, 3], [3, 2, 1]}

GENERATING FUNCTION: $-\frac{x(x^4-2x^3+5x^2-3x+1)}{(x-1)^5}$
sequence to 30 terms: 1, 2, 5, 13, 31, 66, 127, 225, 373, 586, 881, 1277, 1795, 2458, 3291, 4321, 5577, 7090, 8893, 11021, 13511, 16402, 19735, 23553, 27901, 32826, 38377, 44605, 51563, 59306
(not in online encyclopedia of integer sequences)

RECURRENCE: $-\frac{22+28n+13n^2}{38+n}+\frac{(-54-25n+13n^2)N}{38+n}+N^2$
ASYMPTOTIC EXPANSION: $n^a(1-\frac{3876-1047a+13a^2}{26(a-5)n}+\frac{-46512-947086a+398987a^2+169a^4-39806a^3}{2028(a^2+30-11a)n^2})$
ZINN: $a(n)$ asymptotic to $n^{4.046649112}1.001952658^n$
THERE IS 1 SYMMETRY CLASS WITH THIS SEQUENCE:
{[4, 3, 2, 1], [1, 3, 2]}

GENERATING FUNCTION: $-\frac{(x^3-4x^2+3x-1)x}{(2x-1)(x-1)^3}$
sequence to 30 terms: 1, 2, 5, 13, 32, 74, 163, 347, 722, 1480, 3005, 6065, 12196, 24470, 49031, 98167, 196454, 393044, 786241, 1572653, 3145496, 6291202, 12582635, 25165523, 50331322, 100662944, 201326213, 402652777, 805305932, 1610612270
(not in online encyclopedia of integer sequences)
RECURRENCE: $\frac{2(1+n)}{-1+n}-\frac{(-1+3n)N}{-1+n}+N^2$
ASYMPTOTIC EXPANSION: (no result)
ZINN: $a(n)$ asymptotic to $n^{0.3165247290e-5}1.999999266^n$
textscthere is 1 symmetry class with this sequence:
{[1, 2, 3], [3, 2, 4, 1]}

GENERATING FUNCTION: $x(1+2x+25x^5+13x^3+25x^4+5x^2)$
sequence to 30 terms: 1, 2, 5, 13, 25, 25, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
(not in online encyclopedia of integer sequences)
RECURRENCE: $\frac{5(n-6)(89n^2+534n-6095)}{-121410+66337n-14496n^2+1169n^3}+N$
ASYMPTOTIC EXPANSION: $(\frac{-445}{1169})^n n^{(\frac{14496}{1169})}(1+\frac{9504365676}{121623929n}+\frac{42471369406420216616}{14792380105397041n^2})$
ZINN: (no result)
THERE IS 1 SYMMETRY CLASS WITH THIS SEQUENCE:
{[4, 3, 2, 1], [1, 2, 3]}

GENERATING FUNCTION: $-\frac{(2x^2-2x+1)x}{3x^3-5x^2+4x-1}$
sequence to 30 terms: 1, 2, 5, 13, 33, 82, 202, 497, 1224, 3017, 7439, 18343, 45228, 111514, 274945, 677894, 1671393, 4120937, 10160465, 25051354, 61765902, 152288233, 375477484, 925766477, 2282543187, 5627772815, 13875674756, 34211464510, 84350802705, 207972912538
(not in online encyclopedia of integer sequences)
RECURRENCE: $-3+5N-4N^2+N^3$
ASYMPTOTIC EXPANSION: (no result)
ZINN: $a(n)$ asymptotic to $n^{1.200000000e-9}2.465571231^n$
THERE IS 1 SYMMETRY CLASS WITH THIS SEQUENCE:
{[4, 1, 2, 3], [2, 3, 1]}

GENERATING FUNCTION: (n/a)
sequence to 30 terms: 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845, 35357670, 129644790, 477638700, 1767263190, 6564120420, 24466267020, 91482563640, 343059613650, 1289904147324, 4861946401452, 18367353072152, 69533550916004, 263747951750360, 1002242216651368, 3814986502092304
(A005802: Catalan numbers: $C(n) = binomial(2n, n)/(n + 1) = (2n)!/(n!(n + 1)!)$)
(A000108: Catalan numbers: $C(n) = binomial(2n, n)/(n + 1) = (2n)!/(n!(n + 1)!)$)
RECURRENCE: $-\frac{2(1+2n)}{2+n}+N$
ASYMPTOTIC EXPANSION: $\frac{4^n(1-\frac{9}{8n}+145128n^2)}{n^{3/2}}$
ZINN: $a(n)$ asymptotic to $n^{-1.499011783}3.996563696^n$
THERE ARE 10 SYMMETRY CLASSES WITH THIS SEQUENCE:

{[2, 1, 3], [2, 1, 3, 4]}
{[1, 2, 3], [1, 4, 2, 3]}
{[1, 2, 3], [2, 1, 3, 4]}
{[1, 3, 2], [1, 4, 3, 2]}
{[3, 2, 1], [1, 4, 3, 2]}
{[1, 2, 3, 4], [1, 2, 3]}
{[1, 3, 2], [1, 4, 2, 3]}
{[3, 1, 2], [1, 4, 2, 3]}
{[1, 3, 2], [1, 3, 2, 4]}
{[1, 2, 3], [1, 3, 2, 4]}

THERE ARE 8 SYMMETRY CLASSES UNCOUNTED FINLABEL AND WILF.

summarizing results for [4,3] pattern sets
there are 30 symmetry classes in all.
12/30 (40 %) can be counted by FINLABEL.
10/30 (33 %) can be counted by WILF.
thus 73 % of the symmetry classes can be counted by either FINLABEL or WILF.