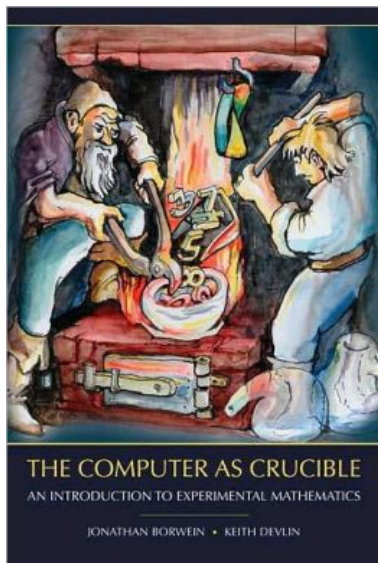


Teaching Inquiry through Experimental Mathematics

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2015 Joint Mathematics Meetings
MAA Session on Teaching Inquiry
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Experimental Math is...



“Experimental math is the use of a computer to **run computations** to **look for patterns**, to **identify** particular **numbers and sequences**, to **gather evidence** in support of specific mathematical assertions that may themselves arise by computational means.”

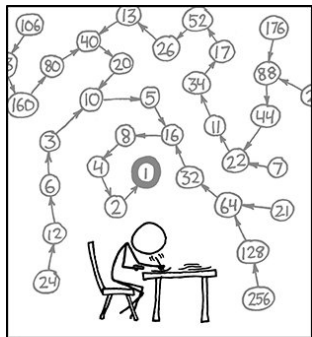
Experimental Math



Experimental Math (Valpo style...)

- ▶ **Throughout the course:**
 - ▶ Mini-essays on philosophy of doing math
 - ▶ Individualized project
- ▶ **Intro: (1.5 weeks)**
 - ▶ What *is* experimental math?
 - ▶ Making friends with the computer
- ▶ **Guided exploration: (11.5 weeks)**
 - ▶ Introduce a new problem
 - ▶ Program together
 - ▶ List of “experiments” in groups
- ▶ **Wrap up (2 weeks):**
 - ▶ Landmarks of computers in proofs
(Four color theorem, Kepler conjecture)
 - ▶ Student showcase

Lesson 1: Collatz Conjecture

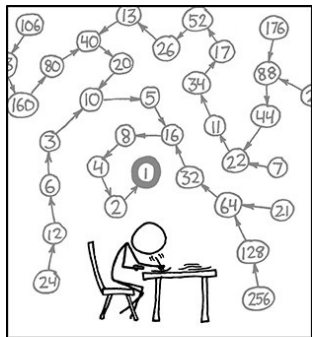


THE COLLATZ CONJECTURE STATES THAT IF YOU PICK A NUMBER, AND IF IT'S EVEN DIVIDE IT BY TWO AND IF IT'S ODD MULTIPLY IT BY THREE AND ADD ONE, AND YOU REPEAT THIS PROCEDURE LONG ENOUGH, EVENTUALLY YOUR FRIENDS WILL STOP CALLING TO SEE IF YOU WANT TO HANG OUT.

Source: <http://xkcd.com/710/>

Lesson 1: Collatz Conjecture

In class...



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$$f(n) = \begin{cases} 3n + 1 & n \text{ odd} \\ n/2 & n \text{ even} \end{cases}$$
- ▶ $g(n, k) = [f(n), f(f(n)), \dots, f^k(n)]$
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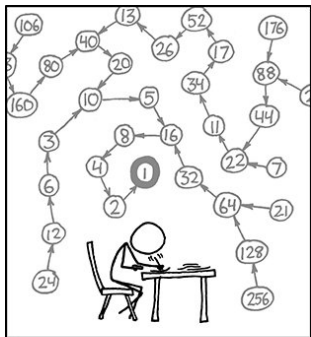
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Experiments...

- ▶ Try out many starting values.

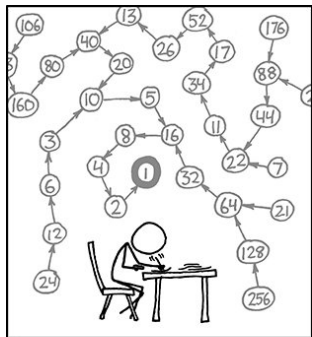


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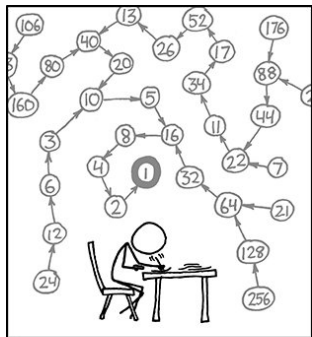
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- ▶ Try out the $5n + 1$ problem.

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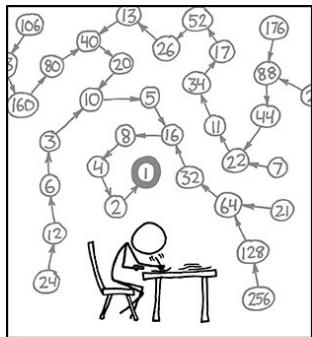
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- ▶ Try out many starting values.
- ▶ Try out the $5n + 1$ problem.
- ▶ Try $f(n) = \begin{cases} 3n + 1 & n \text{ prime} \\ n/p_1 & n = p_1^{e_1} \cdots p_k^{e_k}, p_i < p_{i+1} \forall i \end{cases}$

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- ▶ **Try your own piecewise functions.**

Lesson 2: Integer Relation Algorithms

Algorithm for the Ages: Better Way to Find Integer Relations

Paul Preuss, paul_preuss@lbl.gov

BERKELEY, CA ♦ Among the top ten "Algorithms of the Century" announced in the January/February, 2000, issue of *Computing in Science and Engineering* magazine is the integer-relation algorithm dubbed PSLQ, discovered by mathematician and sculptor Helaman Ferguson of Maryland's Center for Computing Sciences, and implemented in practical computer software by David Bailey, chief technologist of the National Energy Research Scientific Computing Center (NERSC) at the Department of Energy's Lawrence Berkeley National Laboratory.

PSLQ has unearthed many surprising relations in mathematics and physics, although its most startling result may well be a simple formula for calculating any binary digit of pi without calculating the digits preceding it. Before PSLQ, mathematicians had not thought that such a digit-extraction algorithm for pi was possible.

$$\pi = \sum_{k=0}^{\infty} \frac{1}{16^k} \left[\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right]$$

A SIMPLE FORMULA DISCOVERED WITH PSLQ MAKES IT POSSIBLE TO CALCULATE THE NTH BINARY DIGIT OF PI WITHOUT COMPUTING ANY OF THE FIRST N-1 DIGITS, AND DO THE COMPUTATION WITH VERY LITTLE COMPUTING POWER.

<http://www2.lbl.gov/Science-Articles/Archive/pi-algorithm.html>

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- ▶ Implement your algorithm.

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- ▶ Implement your algorithm.
- ▶ Compare your algorithm's results against another group of classmates.

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Experiments...

- ▶ Brainstorm with a classmate to design an integer relation algorithm for 3 real numbers.
- ▶ Implement your algorithm.
- ▶ Compare your algorithm's results against another group of classmates.
- ▶ Compare your algorithm's results against PSLQ (implemented in Maple's IntegerRelations library).

Transitions



= in-class.



= student-generated experiment.

	problem statement	code	data
early classes			
later classes			
final project			




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Reaction

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From an engineering major:

I was always taught: here is a concept, here is what it does, here is how to do it. I figured stuff that I need to learn would always just be given to me. This class has given me an appreciation for actually getting to explore concepts and learn on my own, which is something I would previously never thought would have worked.

Thanks for listening!

Email: Lara.Pudwell@valpo.edu

Slides at: faculty.valpo.edu/lpudwell