The Joy of Discovery

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Mathfest Alder Presentation
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Math is...

- hard
- tedious
- rigid
Math is...

- hard
- tedious
- rigid

or...

- challenging
- elegant
- creative
The plot

Discovery in...

- undergraduate research
- intro courses
- experimental mathematics
Undergraduate Research

Undergraduate research is
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Undergraduate research is *an inquiry or investigation conducted by an undergraduate student that makes an original intellectual or creative contribution to the discipline.* [http://www.cur.org]
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Themed session (Saturday, 8:30-11:25am, 1-3:55pm, Galleria II): *Undergraduate Research in Mathematics: How, When, Why*

Leitzel lecture (Saturday 8:30am, Grand Ballroom): *Research in Mathematics by Undergraduates: Past, Present, and Future*
Once upon a time in precalculus...
Once upon a time in precalculus...

Do I have a vertical asymptote?
Once upon a time in precalculus...
Once upon a time in precalculus...

Am I a graph you’ve seen before?!
The mystery graph revealed

$$f(x)$$

$$(−1, 1)$$  $$ (2.5, 1)$$

$$(−e, 0)$$  $$ (π, 0)$$
The mystery graph revealed

\[ f(x) \]

\((-1, 1) \quad (2.5, 1) \]

\((-e, 0) \quad (\pi, 0) \]

Dirichlet's function?
Lessons learned

- example that breaks the mold
- one activity, many levels
- creative discovery with a “rote” topic
Experimental Math is...

“Experimental math is the use of a computer to run computations to look for patterns, to identify particular numbers and sequences, to gather evidence in support of specific mathematical assertions that may themselves arise by computational means.”
Experimental Math is...
Experimental Math is...

Opinion 72: The Next Term in the Sequence: [Dog, Human, Mathematician, ...] Is "Computer-Programmer for Computer-Generated Mathemachics!"

By Doron Zeilberger
Wates June 11, 2016

I often tell my students that the reason math (especially rigorous math) is so hard is that "mathematicians" is really another species, higher than human sapiens, and a mathematician is to a non-mathematician as a human is to a dog. Did humans and dogs lack one day’s basic skills, such as the understanding of the idea of "the concept" that all of us, in fact, need, but not a mathematician? If so, natural and often often bizarro logic, the dog is not the dog, the dog is not a dog. No, but a mathematician!

I have the highest respect for my dog Zoe, but in some respects it is much superior to me. For example, she can do her little tricks of, well, you know, and I do not have a mathematician! But she is not quite a human, so in some respects our noses, our brains is "better" than her, since the human's nose is bigger.

QUANTA MAGAZINE

In Computers We Trust?
As math grows ever more complex, will computers reign?

> evalb(2*evalb(1/2)-2) 
> true

This simple computation, written with math software called Maple, verifies a number of integer triangles with a given perimeter.

By Natalia B. Weidner
February 1, 2013

Shahriar H. Ekhad, the co-author of several papers in respect to theorems, has been known to prove with a single, succinct computerized theorem and identities that previously required pages of marathons. Last year, when asked to evaluate a formula for the length of integer triangles with a given perimeter, Ekhad performed the calculation less than a second and delivered the verdict: "True."

Shahriar H. Ekhad is a computer. Or, rather, it is any of a family of computer programs used by the mathematician Doron Zeilberger, from a New York office to a supercomputer whose services he occasionally musters. The name — Hebrew for "three bone" — refers to the Ekhad’s earliest incarnation.

"The soul is the software," said Zeilberger, who wrote his own C programming tool called Maple.

A mustached, 63-year-old at Rutgers University and one of the leaders in a growing number of computer-math-phobic groups, Ekhad is one of a number of computer programs that have become so good at math that they are now considering a move to computer science departments.

Focus

An Interview with Doron Zeilberger
By Joe Gallian and Michael Peterson

To celebrate the opening of the Courant House Conference Center at MAA headquarters, made possible by a gift from Paul and Virginia Galbraith, the MAA received a grant from the National Security Agency to support a Distinguished lecturer series intended to appeal to a general audience.

The second lecture in the series was given by Doron Zeilberger on February 20, 2007, in Zeeberger at the Board of Governors Professor of Mathematics at Rutgers University. He is widely known for his development of "WZ (Wilf-Zeilberger) Theory and Zeilberger's algorithm," which are used extensively in modern computer algebra software.

Zeilberger was the first to prove the always-exist combinatorial identity known as the alternating sign matrix conjecture. Among his honors are the AMS Steele Prize for seminal contributions to research, a grant from the National Science Foundation (under which he worked with H. W. Lenstra), the Institute of Combinatorics and its Applications Euler Medal (for "Outstanding Contributions to Combinatorics"), the Knut H. von Neumann Award (for "Advancing the Mathematics of Computational Neuroscience"), and the Lester R. Ford Award for a paper at the American Math.

Special functions are just one source for examples and case studies of a methodology with the aim of making the computer to discover conjectures and then to prove them all by itself, without any human intervention.

1. Are there now a number of people who are doing experimental mathematics and less? Mathematicians through the centuries, using pencil and paper. Of course, with computers you can do so much more, and you can be very systematic and the great power of today's computers, guided wisely, can take you a very long way. However, there is still so much to be found by human intervention...
Experimental Math (Valpo style…)

- **Throughout the course:**
  - Mini-essays on philosophy of doing math
  - Individualized project

- **Intro: (1.5 weeks)**
  - What *is* experimental math?
  - Making friends with the computer

- **Guided exploration: (11.5 weeks)**
  - Introduce a new problem
  - Program together
  - List of “experiments” in groups

- **Wrap up (2 weeks):**
  - Landmarks of computers in proofs
    (Four color theorem, Kepler conjecture)
  - Student showcase
Reaction

From a math major:

I’m learning math isn’t just about memorizing formulas and plugging in numbers, but building on what you know, asking your own questions, and realizing not everything has a known answer just yet.
Reaction

From a math major:

I’m learning math isn’t just about memorizing formulas and plugging in numbers, but building on what you know, asking your own questions, and realizing not everything has a known answer just yet.

From an engineering major:

I was always taught: here is a concept, here is what it does, here is how to do it. I figured stuff that I need to learn would always just be given to me. This class has given me an appreciation for actually getting to explore concepts and learn on my own, which is something I would previously never thought would have worked.
Tedious and rigid, or... elegant and creative?

How (do you/will you) help shift the dialogue?
Tedious and rigid, or... elegant and creative?

How (do you/will you) help shift the dialogue?

- undergraduate research
- experimental math
- student brainstorming and class activities
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