The Joy of Discovery

Lara Pudwell

faculty.valpo.edu/lpudwell

Lara.Pudwell@valpo.edu

Mathfest Alder Presentation

August 8, 2014

Math is...

- hard
- tedious
- rigid

Math is...

- hard
- tedious
- rigid

or...

- challenging
- elegant
- creative

The plot

Discovery in...

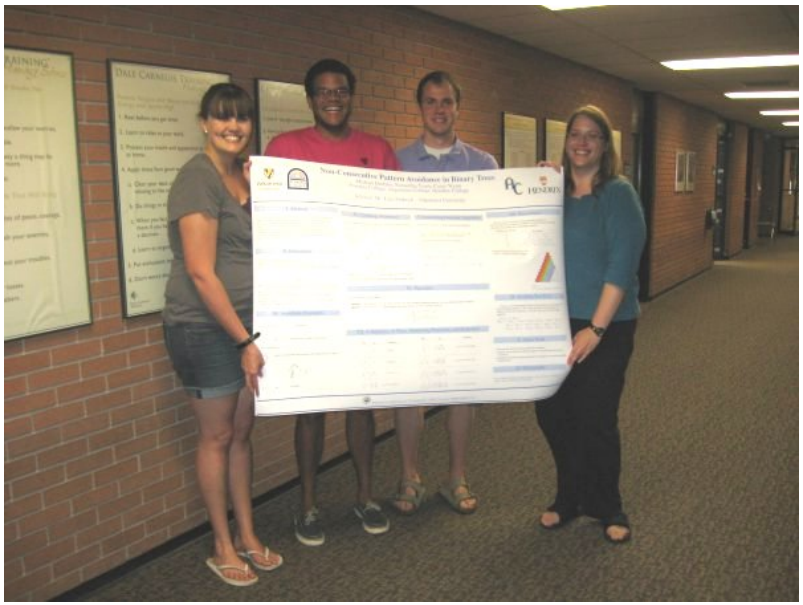
- undergraduate research
- intro courses
- experimental mathematics

Undergraduate Research

Undergraduate research is

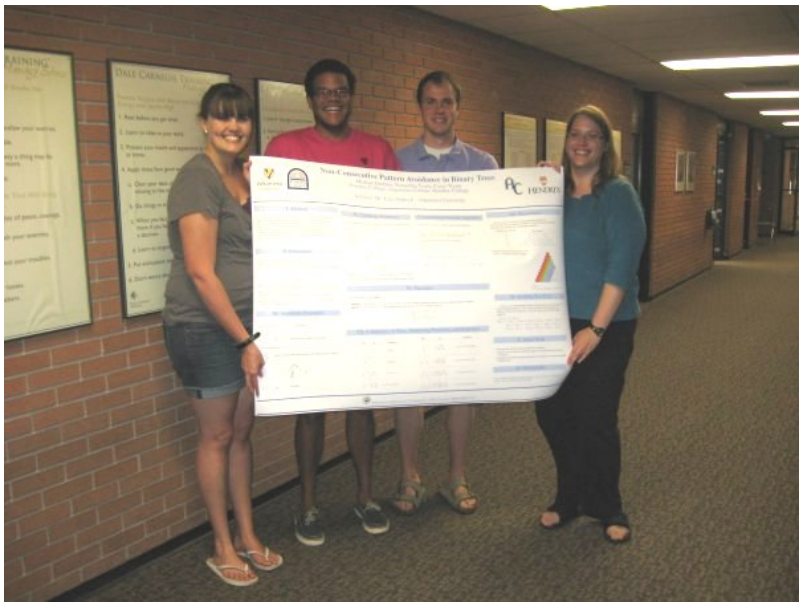
Undergraduate Research

Undergraduate research is *an inquiry or investigation conducted by an undergraduate student that makes an original intellectual or creative contribution to the discipline.* [http://www.cur.org]



Undergraduate Research

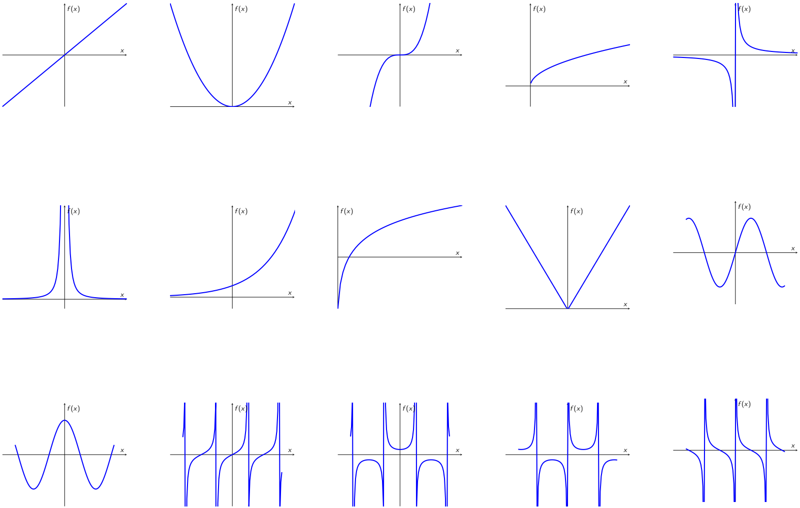
Undergraduate research is *an inquiry or investigation conducted by an undergraduate student that makes an original intellectual or creative contribution to the discipline.* [http://www.cur.org]



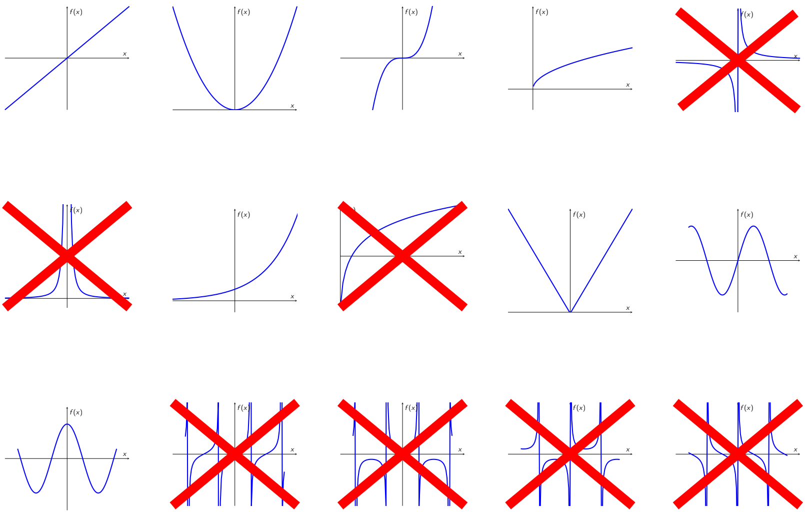
Themed session (Saturday, 8:30-11:25am, 1-3:55pm, Galleria II): *Undergraduate Research in Mathematics: How, When, Why*

Leitzel lecture (Saturday 8:30am, Grand Ballroom): *Research in Mathematics by Undergraduates: Past, Present, and Future*

Once upon a time in precalculus...

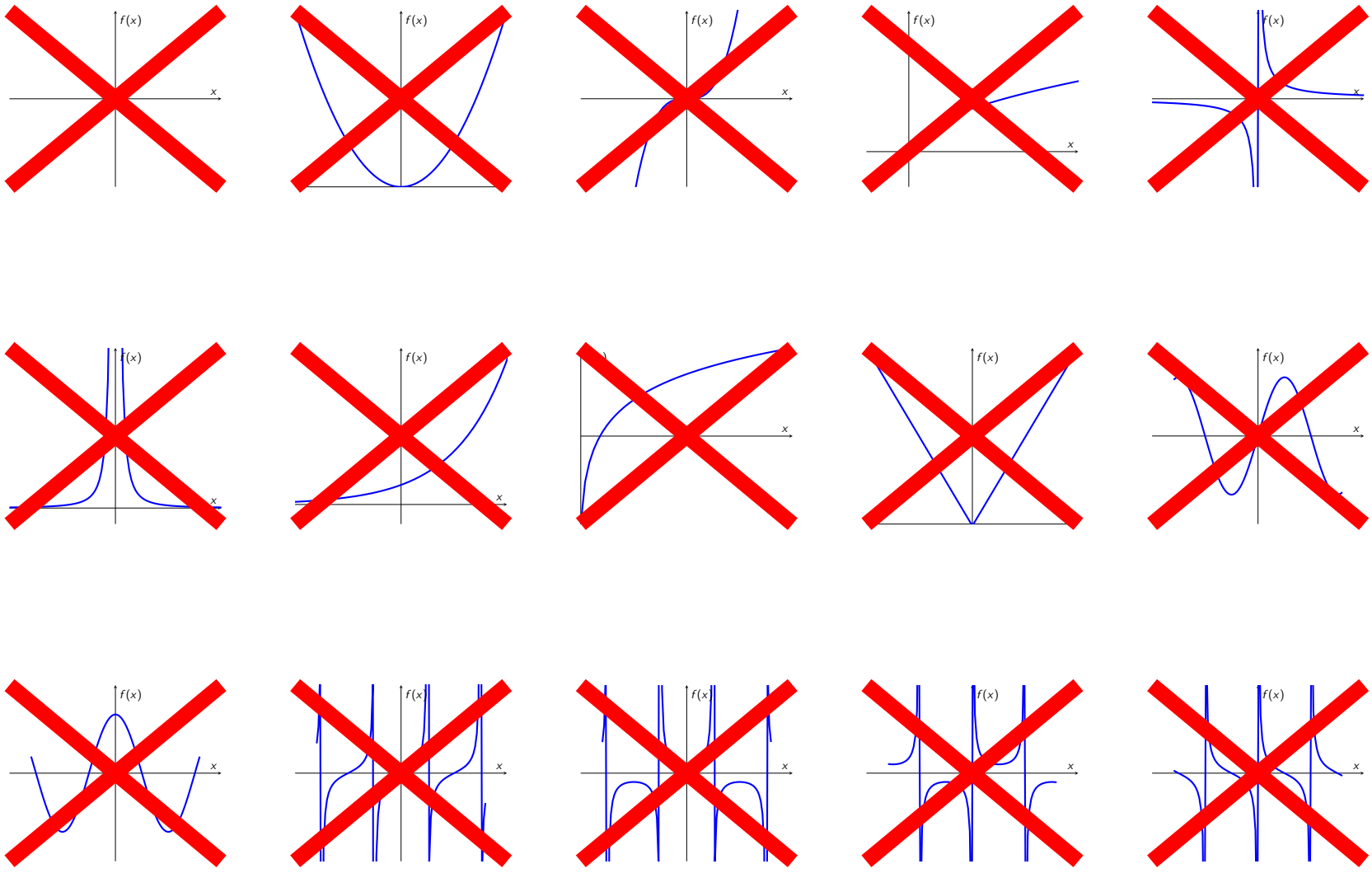


Once upon a time in precalculus...

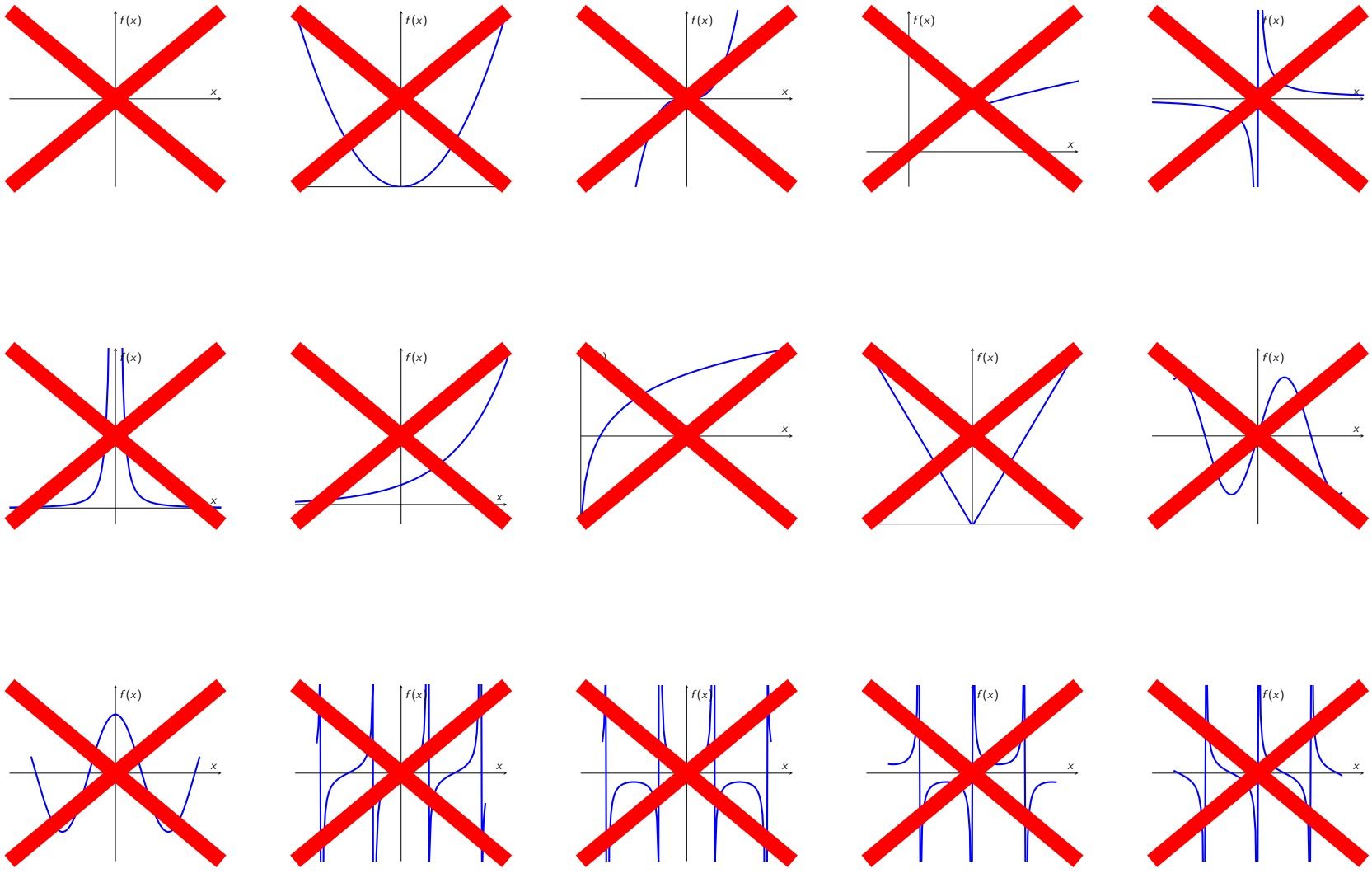


Do I have a vertical asymptote?

Once upon a time in precalculus...

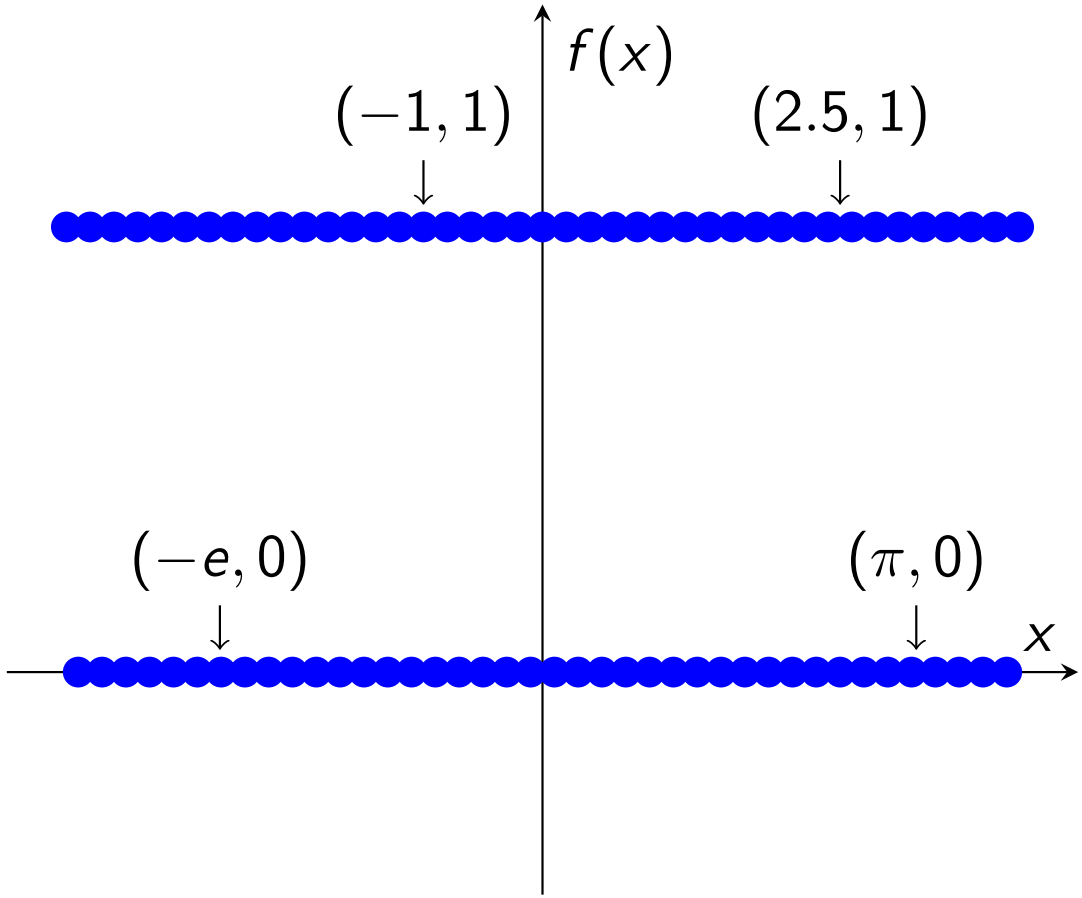


Once upon a time in precalculus...

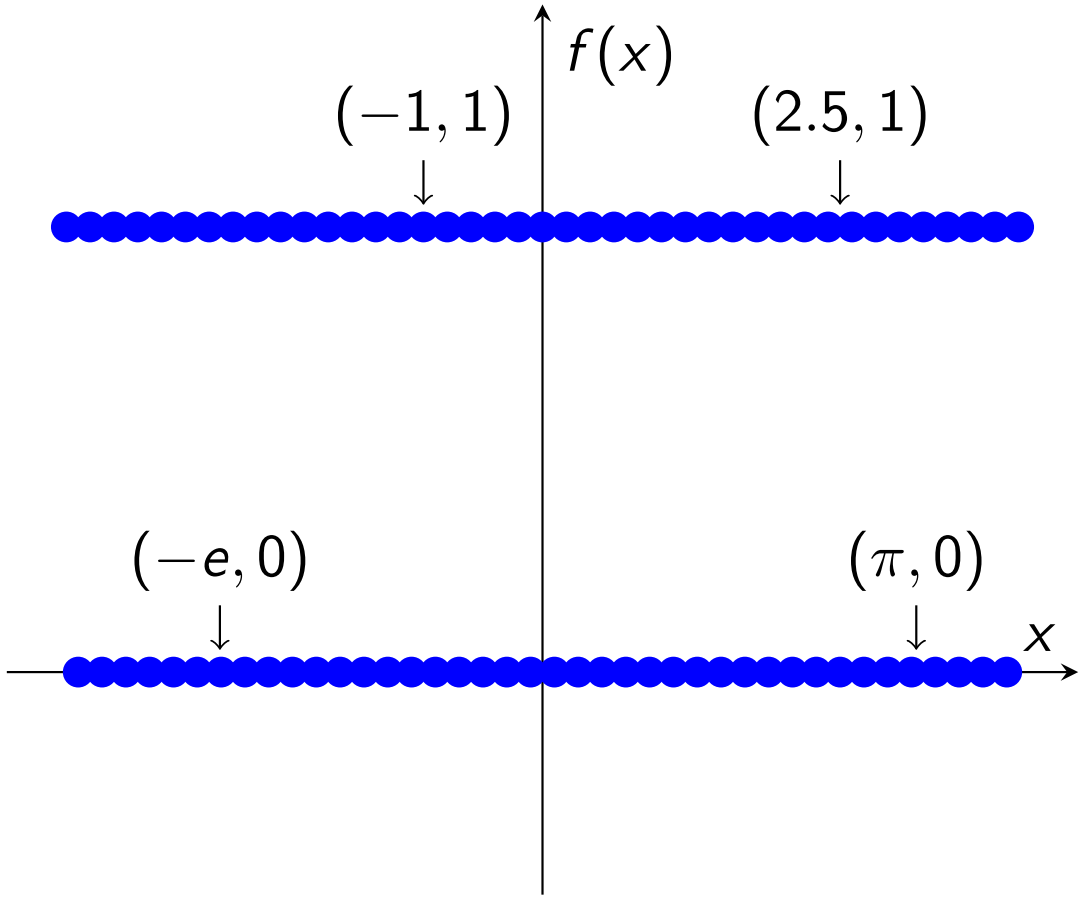


Am I a graph you've seen before?!

The mystery graph revealed



The mystery graph revealed

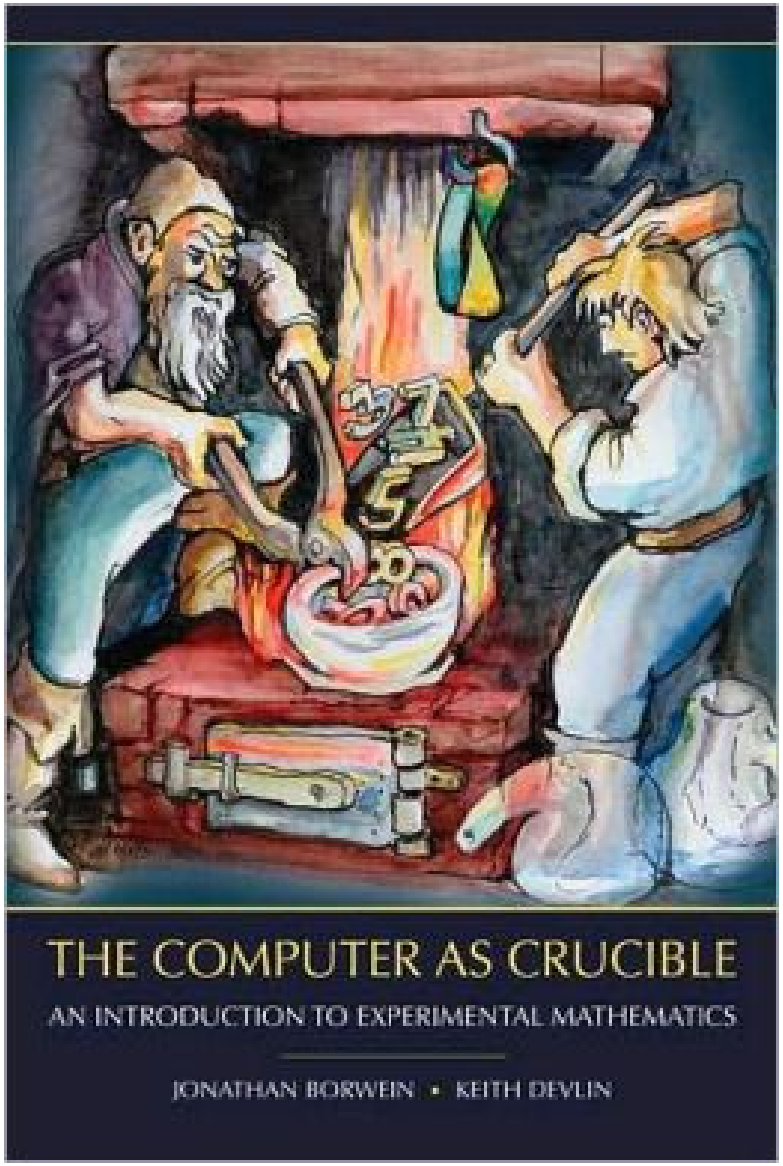


Dirichlet's function?

Lessons learned

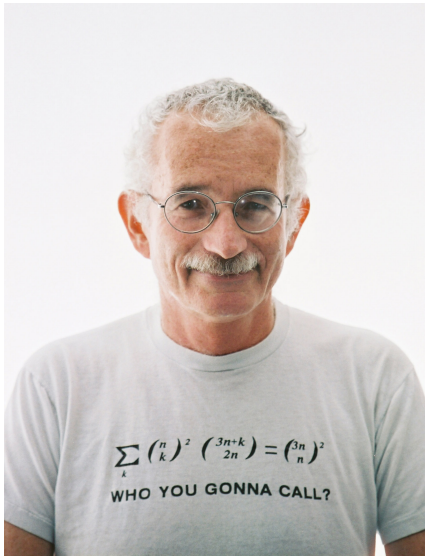
- example that breaks the mold
- one activity, many levels
- creative discovery with a “rote” topic

Experimental Math is...



"Experimental math is the use of a computer to **run computations** to **look for patterns**, to **identify** particular **numbers and sequences**, to **gather evidence** in support of specific mathematical assertions that may themselves arise by computational means."

Experimental Math is...



Experimental Math is...

Opinion 72: The Next Term in the Sequence: [Dog, Human, Mathematician, ...] is ``Computer-Programmer for Computer-Generated Mathematics''

By Doron Zeilberger

Written: June 11, 2006.

I often tell my students that the reason math (especially rigorous math with proofs) is so hard is that "mathematician" is really another species, higher than homo sapiens, and a mathematician is to a non-mathematical human as a human is to a dog. Both humans and dogs have very high cognitive skills, and the abstraction level of a dog far surpasses that of an ant, that in turn, beats that of an amoeba, but for a mathematician, the informal and often flawed logic of the hoipolk is like the logic of a dog compared to that of a human.

I have the highest respect for my dog Zoe, that in many respects is much superior to me. For example, she can run faster and her sense of smell is much better than mine. But she is not quite a human, so in some (rather narrow) sense, any human is "better" than her, since the human's abstraction level is higher.

QUANTA MAGAZINE illuminating science

In Computers We Trust?

As math grows ever more complex, will computers reign?

```
> evalb(seq(coeff(taylor(q^3/(1-q^2)/
(1-q^3)/(1-q^4),q=0,37),q,i),i=0..36)
=seq(round(n^2/12)-trunc(n/4)*trunc((
n+2)/4),n=0..36));

> True
```

This simple computation, written with math software called Maple, verifies a f number of integer triangles with a given perimeter.

By Natalie Wolchover
February 22, 2013

Shalosh B. Ekhad, the co-author of several papers in respecte journals, has been known to prove with a single, succinct ut theorems and identities that previously required pages of mathe reasoning. Last year, when asked to evaluate a formula for the n integer triangles with a given perimeter, Ekhad performed 37 c less than a second and delivered the verdict: "True."

Shalosh B. Ekhad is a computer. Or, rather, it is any of a rotatin computers used by the mathematician Doron Zeilberger, from t New Jersey office to a supercomputer whose services he occasion Austria. The name — Hebrew for "three B one" — refers to the A Ekhad's earliest incarnation.

"The soul is the software," said Zeilberger, who writes his own co popular math programming tool called Maple.

A mustachioed, 62-ye at Rutgers University, anchors one end of a sp opinions about the role in mathematics. He h Ekhad as a co-author the late 1980s "to mal that computers should where credit is due." For decades, he

Opinion

[Contemporary Pure] Math Is Far Less Than the Sum of Its [Too Numerous] Parts

In a newspaper article entitled "Math is more than the sum of its parts" (*New York Daily News*, July 8, 2012), the great pure mathematician Edward Frenkel, along with mathematics educator Ronald Ross, preached the importance of math, apropos of the announcement of the discovery of the Higgs boson.

What Frenkel and Ross did not tell us is that the "math" that led to the discovery of the Higgs boson is not their kind of (pure-and-rigorous) math, but the much more effective, and efficient, nonrigorous mathematics practiced by theoretical physicists called quantum field theory. This highly successful (and precise!) mathematical theory would not be considered mathematics by most members of the American Mathematical Society, since it is com-

their highly technical, usually very dry, preprepared laptop presentations, and (almost) no one has any clue. Indeed, pure math has gotten so splintered that very few people see the mathematical forest. Most can barely understand their own trees.

One example is the AMS Colloquium Lecture series at the Joint Mathematics Meetings. No doubt some of these three-hour lecture series have been very good. But too often they are delivered by talented mathematicians who do not even attempt to make the lectures accessible to a general mathematical audience. Rather, they give highly technical talks with completely unrealistic expectations about the background of the audience.

Mathematics is so useful because physical scientists and engineers have the good sense to largely ignore the "religious" fanaticism of professional mathematicians and their insistence on so-called rigor, which in many cases is misplaced and hypocritical, since it is based on "axioms" that are completely fictional, i.e., those that involve the so-called infinity.

The purpose of mathematical research should be the increase of mathematical knowledge, broadly defined. We should not be tied up with the antiquated notions of alleged "rigor". A new philosophy of and attitude toward mathematics is developing, called "experimental math"

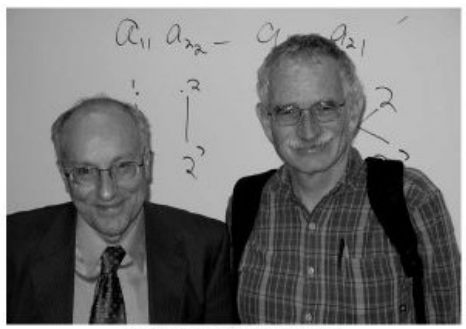
FOCUS **May/June 2007**

An Interview with Doron Zeilberger

By Joe Gallian and Michael Pearson

To celebrate the opening of the Carriage House Conference Center at MAA headquarters, made possible by a gift from Paul and Virgina Halmos, the MAA received a grant from the National Security Agency to support a Distinguished Lecture series intended to appeal to a general audience.

The second lecture in the series was given by Doron Zeilberger on February 20, 2007. Zeilberger is the Board of Governors Professor of Mathematics at Rutgers University. He is widely known for the development of "WZ" (Wilf-Zeilberger) Theory and Zeilberger's algorithm which are used extensively in modern computer algebra software. Zeilberger was the first to prove the elusive result in combinatorial theory known as the alternating sign matrix conjecture. Among his honors are: the American Mathematical Society Steele Prize for seminal contributions to research (co-recipient with Herb Wilf); the Institute of Combinatorics and Its Applications Euler Medal for "Outstanding Contributions to Combinatorics"; the Laura H. Carnell Professorship at Temple University; and the MAA Lester R. Ford award for a paper in *The American Math-*



Joe Gallian and Doron Zeilberger

special functions are just sources for examples and case studies of a methodology with the aim of training the computer to discover conjectures and then try to prove them all by itself, without any human intervention.

JG: Are there now a number of people who are doing experimental mathematics?

and less-great, mathematicians through the centuries, using pencil-and-paper. Of course, with computers you can do so much more, and you can be very systematic, and the great power of today's computers, guided wisely, can take you a very long way. However, their emphasis is still on using computers to find interesting conjectures and phenomena, but not to prove them. The proof itself (when feasible) is still done largely by human

Experimental Math

Experimental Math (Valpo style...)

- **Throughout the course:**
  - Mini-essays on philosophy of doing math
  - Individualized project
- **Intro: (1.5 weeks)**
  - What *is* experimental math?
  - Making friends with the computer
- **Guided exploration: (11.5 weeks)**
  - Introduce a new problem
  - Program together
  - List of "experiments" in groups
- **Wrap up (2 weeks):**
  - Landmarks of computers in proofs
    (Four color theorem, Kepler conjecture)
  - Student showcase

Reaction

From a math major:

> *I'm learning math isn't just about memorizing formulas and plugging in numbers, but building on what you know, asking your own questions, and realizing not everything has a known answer just yet.*

Reaction

From a math major:

I'm learning math isn't just about memorizing formulas and plugging in numbers, but building on what you know, asking your own questions, and realizing not everything has a known answer just yet.

From an engineering major:

I was always taught: here is a concept, here is what it does, here is how to do it. I figured stuff that I need to learn would always just be given to me. This class has given me an appreciation for actually getting to explore concepts and learn on my own, which is something I would previously never thought would have worked.

Tedious and rigid, or... elegant and creative?

How (do you/will you) help shift the dialogue?

Tedious and rigid, or... elegant and creative?

How (do you/will you) help shift the dialogue?

- undergraduate research
- experimental math
- student brainstorming and class activities

Tedious and rigid, or... elegant and creative?

How (do you/will you) help shift the dialogue?

- undergraduate research
- experimental math
- student brainstorming and class activities

Email: `Lara.Pudwell@valpo.edu`
Slides at: `faculty.valpo.edu/lpudwell`