What is a mathematician?

Lara Pudwell
Assistant Professor of Mathematics and Computer Science
Valparaiso University

Discoveries Unlimited Speakers Bureau
October 18, 2012
What is a mathematician?
Lara Pudwell

What do I do?
Teaching
Service
Research

How did I get here?
High School
College
Graduate School
...and beyond

Other Mathematical Careers

Conclusion
Recap
Recommended Reading
What does a mathematician do?

At Valparaiso University, my job is

- 50% teaching
- 30% research
- 20% service

...but what does this mean?
What is a mathematician?
Lara Pudwell

What do I do?
Teaching
Service
Research

How did I get here?
High School
College
Graduate School
...and beyond

Other
Mathematical Careers

Conclusion
Recap
Recommended Reading
Teaching

- teach students in class.
- grade student work.
- prepare lesson plans and class activities.
- answer student questions in office hours.
What is a mathematician?

Lara Pudwell

What do I do?

Teaching
Service
Research

How did I get here?

High School
College
Graduate School
...and beyond

Other
Mathematical
Careers

Conclusion
Recap
Recommended
Reading

Service

participate in Valparaiso University committees.
advise students on what classes to take.
write math exams and contests for other schools.
edit/review math articles for journals.
Service

- participate in Valparaiso University committees.
- advise students on what classes to take.
- write math exams and contests for other schools.
- edit/review math articles for journals.
What is a mathematician?
Lara Pudwell

What do I do?
Teaching
Service
Research

How did I get here?
High School
College
Graduate School
...and beyond

Other
Mathematical Careers

Conclusion
Recap
Recommended Reading

Research

Research

solve math problems that no one has ever solved before.
collaborate with students to solve math problems.
attend conferences to work with other mathematicians and present my research results.
Research

- solve math problems that no one has ever solved before.
- collaborate with students to solve math problems.
- attend conferences to work with other mathematicians and present my research results.
What is math research anyhow?

- Conjecture/discover patterns/solutions.
- Write a “proof” of why a pattern/solution works.
- Mathematicians like to ask “Why?”
What is math research anyhow?

- Conjecture/discover patterns/solutions.
- Write a “proof” of why a pattern/solution works.
- Mathematicians like to ask “Why?”
- Proof example: The checkerboard problem.
  Can you cover these checkerboards with non-overlapping dominoes?

![Checkerboard](image)

Why or why not?
What is math research anyhow?

- Conjecture/discover patterns/solutions.
- Write a “proof” of why a pattern/solution works.
- Mathematicians like to ask “Why?”

Proof example: The checkerboard problem.
Can you cover these checkerboards with non-overlapping dominoes?

Why or why not?
The nine dots problem.
Can you draw 4 straight lines that go through all the dots without lifting your pencil?
We’ll say a “word” is a list of numbers. (examples: 01011, 1204252, etc.)

- A double in a word is two copies of the same smaller word back to back. (examples: 021221234, 0211312)
- Question: If you only use 0s and 1s, what’s the longest word you can make without a double?
We’ll say a “word” is a list of numbers. (examples: 01011, 1204252, etc.)

- A *double* in a word is two copies of the same smaller word back to back. (examples: 02121234, 0211312)
- Question: If you only use 0s and 1s, what’s the longest word you can make without a double?
  - Answer: 010 or 101
We’ll say a “word” is a list of numbers. (examples: 01011, 1204252, etc.)

- A *double* in a word is two copies of the same smaller word back to back. (examples: 021221234, 0211312)
- Question: If you only use 0s and 1s, what’s the longest word you can make without a double?
  - Answer: 010 or 101

- Question: If you only use 0s, 1s, and 2s, what’s the longest word you can make without a double?
A taste of research

We’ll say a “word” is a list of numbers. (examples: 01011, 1204252, etc.)

- A *double* in a word is two copies of the same smaller word back to back. (examples: 021221234, 0211312)

- Question: If you only use 0s and 1s, what’s the longest word you can make without a double?
  - Answer: 010 or 101

- Question: If you only use 0s, 1s, and 2s, what’s the longest word you can make without a double?

- Question: Start with 0. Then, append the smallest number possible without making a double. Repeat. Describe the pattern that you find!
Question: If you only use 0s, 1s, and 2s, what’s the longest word you can make without a double?

- Answer: There are infinitely long double-free words with 0s, 1s, and 2s. (proved by Axel Thue in 1906).

If you want to create a double-free word without getting stuck,

1. Start with 012
2. Replace every 0 with 012.
3. Replace every 1 with 02.
4. Replace every 2 with 1.
5. Repeat!
Question: If you only use 0s, 1s, and 2s, what’s the longest word you can make without a double?

Answer: There are infinitely long double-free words with 0s, 1s, and 2s. (proved by Axel Thue in 1906).

If you want to create a double-free word without getting stuck,

1. Start with 012
2. Replace every 0 with 012.
3. Replace every 1 with 02.
4. Replace every 2 with 1.
5. Repeat!

Example: 012 → 012021 → 012021012102 → 012021012102012021020121 → ...
More Patterns Revealed!

Question: Start with 0. Then, append the smallest number possible without making a double. Repeat. Describe the pattern that you find!

Answer: 0102010301020104010201030102010...
Question: Start with 0. Then, append the smallest number possible without making a double. Repeat. Describe the pattern that you find!

Answer: 0102010301020104010201030102010...

- Every other number is 0.
More Patterns Revealed!

Question: Start with 0. Then, append the smallest number possible without making a double. Repeat. Describe the pattern that you find!

- Answer: 0102010301020104010201030102010...
- Every other number is 0.
- If we look at the nonzero numbers: 121312141213121...
More Patterns Revealed!

Question: Start with 0. Then, append the smallest number possible without making a double. Repeat. Describe the pattern that you find!

- Answer: 0102010301020104010201030102010...
  - Every other number is 0.
  - If we look at the nonzero numbers: 121312141213121...
  - ...and subtract 1 from all of them 010201030102010...
More Patterns Revealed!

Question: Start with 0. Then, append the smallest number possible without making a double. Repeat. Describe the pattern that you find!

- **Answer:** 0102010301020104010201030102010...
  - Every other number is 0.
  - If we look at the nonzero numbers: 121312141213121...
  - ...and subtract 1 from all of them 010201030102010...
  - we get back the original word!
More Patterns Revealed!

Question: Start with 0. Then, append the smallest number possible without making a double. Repeat. Describe the pattern that you find!

- Answer: 0102010301020104010201030102010...
  - Every other number is 0.
  - If we look at the nonzero numbers: 121312141213121...
  - ...and subtract 1 from all of them 010201030102010...
  - we get back the original word!

- In fact, if we factor every integer...
More Patterns Revealed!

Question: Start with 0. Then, append the smallest number possible without making a double. Repeat. Describe the pattern that you find!

- Answer: 0102010301020104010201030102010...
  - Every other number is 0.
  - If we look at the nonzero numbers: 121312141213121...
  - ...and subtract 1 from all of them 010201030102010...
  - we get back the original word!

- In fact, if we factor every integer...
  - then the $i$th number in our word is the number of 2s in the factorization of $i$!
Question: Start with 0. Then, append the smallest number possible without making a double. Repeat. Describe the pattern that you find!

- Answer: 0102010301020104010201030102010...
  - Every other number is 0.
  - If we look at the nonzero numbers: 121312141213121...
  - ...and subtract 1 from all of them 010201030102010...
  - we get back the original word!

- In fact, if we factor every integer...
- then the $i$th number in our word is the number of 2s in the factorization of $i$!
- (proved by Guay-Paquet and Shallit in 2009.)
Areas of Mathematical Research

- Algebra – structure of numbers
- Analysis – rates of change
- Combinatorics – counting, optimal arrangements
- Geometry/Topology – shape and symmetry
- Applied Mathematics – probability, statistics, computational math, math for other sciences
How did I get here?
Grade School/High School

- Took honors and AP math classes (geometry, algebra 1 and 2, precalculus, AP calculus BC).
- Participated in school, city, state, and national math contests.
- Participated in regional math team.
- Member of Mu Alpha Theta (national math honors society).
- Tutored other students in math classes.
B.S. in mathematics and B.A. in computer science.
student grader for various math classes.
was paid to participate in a summer research program.
undergraduate research – published 4 papers!
studied abroad in Budapest, Hungary.
### Math
- Abstract Algebra 1 & 2
- Analysis 1 & 2
- Calculus 1, 2, & 3
- Combinatorics
- Differential Equations
- Discrete Math
- Graph Theory
- Linear Algebra
- Number Theory
- Research in Mathematics
- Statistics
- Topology

### Computer Science
- Analysis of Algorithms
- Artificial Intelligence
- Computer Architecture
- Database Management Systems
- Introduction to Programming 1, 2, & 3
- Organization of Programming Languages
- Theory of Computation
Ph.D. in mathematics (5 years of classes and research after college).
- took advanced classes that go beyond college-level math.
- discovered and proved new mathematical facts to write a Ph.D. thesis.
- taught undergraduate students.
- got paid an annual stipend to be in graduate school.
Assistant Professor at Valparaiso University since 2008.

Next: Associate Professor, then Full Professor.

continue to solve problems and study new math ideas, and teach math to others.

run summer research program for undergraduate students at Valpo.

travel to conferences to talk about research with other mathematicians.
Other Mathematical Careers
Government-employed mathematicians

work at

- national labs.
- NASA.
- National Security Agency. (largest employer of mathematicians in the country!)
  - NSA internships exist for college students.
Company-employed mathematicians

Some companies have entire mathematics departments or hire lots of mathematicians to solve problems or build mathematical models.

- IBM (computers)
- AT&T
- Telcordia (phones)
- Epic (medical software)
- Amazon.com
- Google
- Pixar
- banks and investment companies (finance industry)
- ... and many more!
Operations Research Specialists

帮公司更高效地运营。

使用统计、建模和优化方法来帮助公司做出更好的决策。

需要强大的数学背景以及商业知识。
Actuaries

- assess risk for insurance and other industries.
- typically major in actuarial science.
- after college, take actuarial exams.
- require strong background in math as well as business, economics, and finance.
Really, math can be combined with just about any other area to make you even more successful!

Recent Valpo student examples:

- Math + art = art conservationist
- Math + political science = policy analyst
- Math + geography = urban planner
Conclusion
Recap

- There’s more variety to being a professor than just teaching.
- There are many unsolved math questions just waiting to be explored!
- Math research can be pure or applied.
- Most careers in math require at least a bachelor’s degree, some require a Masters, or Ph.D.
- Math can be combined with just about any other interest to make you an excellent candidate for whatever career you want.
Recommended Reading

We Use Math In Careers

According to the Wall Street Journal:
Mathematician is considered the Best Job of All Jobs. Mathematicians median annual income was pegged at $94,160.
The top 3 jobs on the list were:
Recommended Reading

- 101 Careers in Mathematics edited by Andrew Sterrett

- Letters to a Young Mathematician by Ian Stewart

- She Does Math! edited by Marla Parker