Successfully Mentoring Undergraduates in Research:
A How To Guide for Mathematicians

By Michael Dorff, Allison Henrich, and Lara Pudwell

Doing research with undergraduates can be one of the most satisfying aspects of our jobs as math professors. For those of us who love both teaching and doing research, what could be better than mentoring and collaborating with students? The three of us certainly feel that working with undergraduates on research is incredibly rewarding! Collectively, we've mentored over 100 students in research, helping nearly all of these students give talks or posters on their work, and publishing papers with many of our research students. Also, we have been involved with the Council on Undergraduate Research (CUR), a leading voice for undergraduate research in all disciplines. In addition, through the Center for Undergraduate Research in Mathematics (CURM) and the Preparation for Industrial Careers in Mathematical Sciences (PIC Math) programs, Michael has worked with over 150 faculty members through summer faculty training workshops to help professors in their efforts in mentoring over 1000 undergraduate students in research.

We've learned through our experience that there are several things to consider as you get started in undergraduate research, whether you are just considering starting your first project with a student, having only worked with graduate students or peers on research until now, or you already have some experience with undergraduate research. To help you prepare for your next research experience with students, we will discuss the Six Fundamental Steps needed to successfully mentor students in research.

The **Six Fundamental Steps** are:
- (1) Picking an appropriate research problem
- (2) Recruiting and selecting students to mentor
- (3) Setting expectations and dealing with group dynamics
- (4) Starting the research and moving it forward
- (5) Helping students develop communication skills
- (6) Preparing for the future

Before we discuss these Six Fundamental Steps in depth, it makes sense to define both *undergraduate research* and *success*. The Council on Undergraduate Research (CUR) defines undergraduate research as "an inquiry or investigation conducted by an undergraduate student that makes an original intellectual or creative contribution to the discipline" [CUR 2015]. We, the authors, agree that undergraduate research involves students working on problems that *no one* has solved before, although some people think of undergraduate research simply as students working on problems that *the students* have never solved before. One reason that we prefer the CUR definition is that, in talking with recruiters from over 50 companies that hire math majors, we have heard them say many times that they are looking for students who have worked for an extended period of time on an unsolved problem (e.g., in an undergraduate research experience or an internship). Companies look to hire students who have had this type of experience because these unsolved problems are precisely the types of problems their employees work on [Dor

2014]. On the other hand, while undergraduate research is similar in many respects to the kind of research you might do with peers, it is good to understand that there are often differences in the way these two sorts of research are conducted. People who claim that you cannot do undergraduate research in mathematics usually have not yet come to this understanding. These distinctions will become apparent in the discussion that follows, but for additional references detailing the unique benefits of undergraduate research, see [Dor 2014].

We would also like to recognize that what each of us would consider a "successful" undergraduate research experience may be different. So we encourage you to ask yourself: "What is the reason I am doing undergraduate research?" and "What expectations do I have of my students and of the experience as a whole?" Your answers to these questions will help you determine what success looks like for you and will influence how you approach the Six Steps. Do you want to mentor undergraduate students to further your own research? If so, do you expect to get a publication from mentoring these students, and what type of journal are you aiming for? Do you want to do undergraduate research because it is one of the best methods for teaching mathematical concepts? Do you want to use it as a way to help prepare students for graduate school or non-academic careers? Before we move on, we should be clear that these are all perfectly good reasons for doing undergraduate research, contributing to valid definitions of success.

Let's see how your answers to these questions could affect your approach to mentoring. Suppose you plan to do undergraduate research as a way to strengthen your own research record. This would naturally influence the types of research problems you would choose for your students. You would still need to pick research problems that are appropriate for your students' level of expertise, of course, but these problems would likely be related to your own research program. Students could create examples demonstrating a theorem you have proved or they could take a theorem you have proved and try to apply it to another situation. Let's consider an example of Michael's. In complex analysis, he once proved a theorem that involved operating the function $\boldsymbol{f(z) = z}$ on a family of analytic functions. The function $\boldsymbol{F\,(\mathrm{z}) = (z - \overline{a})/(1 - \overline{a}\,z)}$ where $\boldsymbol{a} \in |\boldsymbol{z}| < \boldsymbol{1}$ is a generalization of the function $\boldsymbol{f(z) = z}$ (i.e., $\boldsymbol{f = F}$ when $\boldsymbol{a = 0}$). Michael had some students formulate an equivalent theorem using $\boldsymbol{F}$ instead of $\boldsymbol{f}$, and then had them use the pattern of the original theorem with $\boldsymbol{f}$ to prove the equivalent theorem for $\boldsymbol{F}$.

In addition to the types of problems you choose to work on, the selection of the students to mentor would be influenced by your goal of working on a problem related to your own research. You would probably look first for more advanced undergraduates who may have already taken a first course in your research area such as algebra, number theory, topology, or complex analysis. On the other hand, suppose your decision to do undergraduate research is inspired by a desire to prepare students for graduate school by giving them a taste of what math research is like. This would open up more possibilities for research topics. For instance, Allison enjoys studying games that can be played on knot diagrams with students. This kind of research is fun and requires little background knowledge on the students' part. She invited several of her research students to invent a new game to be played on knot diagrams and asked them to think about winning strategies for certain starting positions. The goal wasn't to get a published paper, but to give the students experience with asking questions and developing a plan to answer their questions. Some answers required students to learn some game theory, some questions required knot theory to answer, and some questions could be answered by writing a computer program. In this general scenario, questions might be accessible to less advanced students. This type of research project can provide a great opportunity to work with students who are uncertain about continuing in mathematics or to encourage good, but less confident students.

As Gillman and Szaniszlo state in their description of academic-year undergraduate research at Valparaiso University [GS 2007], the goals of a successful undergraduate research experience are that the project "models the research experience of mathematicians, provides a growth experience appropriate for the maturity level of the participating student, helps students build meaningful connections to the faculty and the department, and introduces students early in their studies to the discovery of new mathematics." A successful undergraduate research experience focuses on student growth and allows the student(s) to interact with material at a level that goes beyond what they do in the classroom; it allows them to develop their own conjectures and proofs.

Keeping these ideas in mind, let's discuss the Six Fundamental Steps needed to successfully mentor students in research. Aspects of these steps are also discussed in articles such as [BBDG 2009], [Cal 2014], [Das 2013], [Die 2013], [EMP 2013], [GS 2007], [Leo 2008], [MV 2014], [Rob 2013].

<u>Step 1: Picking an appropriate research problem</u>

A common difficulty for faculty who are just starting to mentor undergraduate students in research is to choose an appropriate research problem. Your choice of problem should be informed both by your goals for the project and by the students' background. A problem that fits into your current research agenda will look different than a problem chosen with the main goal of leaving room for students to make their own conjectures. Regardless of your personal goals for the research, it is important to consider the level of the students' background when choosing a problem. It is typical for beginning faculty mentors to pick a research problem that is too difficult for students to finish. How can you avoid this pitfall, especially if you don't have a lot of experience in working with undergraduates on research? Our friend Kathryn Leonard talks about how she chooses an appropriate research problem by thinking of it as a problem she could solve on a lazy afternoon.

Some qualities of an appropriate research problem for students to work on are: (1) the problem should require a limited amount of background material, (2) it should be specific and concrete, (3) it should lend itself to creating specific examples and may include using computers, (4) it should have multiple layers starting out somewhat simple and then progressing into higher levels of difficulty, (5) it should be of interest to you and your research community, and (6) you should have some idea of how the problem might be solved. A problem with multiple layers is the opposite of an all-or-nothing problem in which the students have to prove a result and if they don't, then they have nothing to show for their efforts. A recent article about PRIMES (Program for Research In Mathematics, Engineering, and Science) which is a mathematics research program at MIT for high school students also describes many of these components for choosing an appropriate research problem [EGK 2015].

As a basic example, consider the following problem. The area of a circle of radius *r* is $\boldsymbol{A(r) = \pi r^2}$ and the perimeter is $\boldsymbol{P(r) = 2\pi r}$. So, $\boldsymbol{dA/dr = P}$. For what other geometric shapes does this derivative equation hold? If we look at a square with side length $\boldsymbol{x}$, then $\boldsymbol{A(x) = x^2}$ and $\boldsymbol{P(x) = 4x}$, and $\boldsymbol{dA/dx \neq P}$. However, if we inscribe a circle of radius $\boldsymbol{r}$ into the square, then we can write $\boldsymbol{A(r) = (2r)^2}$ and $\boldsymbol{P(r) = 8r}$ and $\boldsymbol{dA/dr = P}$. What other geometric shapes satisfy $\boldsymbol{dA/dr = P}$ and what relationship does $\boldsymbol{r}$ have in these shapes? Some results are known about what other geometric shapes satisfy $\boldsymbol{dA/dr = P}$, but there are more problems to explore (see [DH 2003]). Do you see how this problem has many of the qualities of an appropriate problem for undergraduate students?

If you can't think of a research problem from your own field that is appropriate for undergraduate students, there are various resources to help you find a problem. These resources include journals that publish undergraduate-level work with easily extendable results such as *Involve*, *SIAM Undergraduate Research Online*, *MACE Journal*, or *UMAP Journal*. Conference sessions in which students present their research, such as at the MAA's MathFest, the Joint Mathematics Meetings, or local MAA sectional meetings, can also be sources of inspiration. If you read an interesting paper or see a talk on work done by undergraduates, you can reach out to the project's research mentor to find out which questions related to their project have yet to be solved and which of those they don't expect to work on themselves. In addition, you may consider online resources such as openproblemgarden.org, which lists problems and recommends certain problems as suitable for undergraduates.

Finally, realize that a little creativity on your part may pay off. Maybe the question you ultimately explore with students is not one you found verbatim from one of these resources, but rather your own twist, by modifying a key definition in the problem you first hear. For example, Lara once attended a conference talk on pattern avoidance in graph-theoretic trees. The speaker's definition was that tree **T** contains tree **t** as a pattern if **t** is a subgraph of **T**. Lara proposed an alternate definition of tree pattern involving edge contractions. The speaker had not thought of this definition and encouraged her to pursue it. That modified definition fueled two successful summer REU projects where her students took ownership of the new definition, and their work led to cited publications in recognized combinatorics journals.

Step 2: Recruiting and selecting students to mentor

Recruiting and selecting students to work with you is Step 2. Many students do not realize there are undergraduate research opportunities, or if they do, many assume they are not qualified. Alayont, et. al. discuss how to combat these perceptions in [ABJS 2014]. Since students will not know to come to you for a research opportunity, you need to go to them!

There are various ways to recruit students. You can find students in classes you have taught, ask colleagues to recommend students, or send out an email to all the math majors asking for students interested in doing a research project with you. Some students may be naturally drawn to you because they were in a class you taught and enjoyed it.

To recruit students, Allison likes giving a talk in her university's Undergraduate Mathematics Colloquium when she needs to drum up students. A few years ago, she gave a talk on pseudodiagrams of knots and knot games to students in her department. Afterwards, three students immediately asked her if they could work with her on research. This led to a two-year long project with these three students that didn't result in a publication, but taught the students valuable research skills and helped them figure out whether or not they wanted to go to graduate school.

Michael has found success with giving a short description or presentation of his research that he can give to students to capture their interest. Michael's research is in complex-valued harmonic mappings that are a generalization of analytic functions, and he studies properties that are preserved under convolutions. When he tries to recruit students to do research, do you think that's how he explains his research? Of course not! Instead, he talks about creating soap films by dipping wire frames into soap solution. He explains to students that he studies the properties of these soap films. In fact, almost every semester he will bring in a bucket of soap solution with some wire frames one day during class and demonstrate some soap films. Students are enthralled and it is common for a student to contact him later asking him about doing undergraduate research. (You may be wondering, what is the connection between Michael's research and dipping wire frames in soap solutions? The answer is that under certain conditions, complex-valued harmonic functions lift to minimal surfaces, and minimal surfaces can be modeled by soap films.)

Current research students can be the most effective recruiters for potential new students for a research group. For instance, Allison and her research student Christopher went to a Mariners baseball game with several other faculty and students from their math department. They gave a younger student named Colin a ride to the game, so they all had some time to chat. Christopher spent most of the ride explaining his research to Colin. A few weeks later, Colin showed up in Allison's office asking if he could join the research group. Christopher and Colin were a great research team for the next year. They gave a fascinating talk on their results at a local MAA sectional meeting at the end of the school year, and Colin continued working on a related research problem for a second year with Allison after Christopher graduated.

Most professors start doing undergraduate research by working with one student on one problem. Over time, though, many discover that it's more efficient to have a research group of 2-5 students working on the same problem at the same time. The three of us generally try to set up our research groups so that we have some advanced students and some beginning students. The advanced students are students who have worked with us longer on research and who have more math background. By having a group with different levels of experience, you can have the more advanced students help you mentor the beginning students in some of the basic ideas and procedures for your research. This mentoring experience is great for the advanced students, and it takes less of your time – this has really helped us to continue to do undergraduate research even as the demands on our time have increased. It's a sustainable model. When the advanced students graduate, beginning students move up to the role of advanced student, and they can recruit new students to become the beginning students. Your own research students know what the research entails and usually have a better grasp than you do on the suitability and compatibility of their peers. Also, your research students will typically recruit students they get along with, so you'll have fewer issues with group dynamics to sort out.

Once you have a system for recruiting students, you should decide how to select students to actually do research. Often, faculty members want students to have a minimum amount of mathematical background. If you are doing research in mathematical biology you might require students to have a differential equations course under their belts. If your research involves proving theorems, you might want your students to have an Introduction to Proofs course. We have found that the most important quality for an undergraduate researcher to have is that she/he is a hard worker who is eager to do research. This is more important, in our experience, than intelligence or level of mathematical background.

Michael's research area is complex analysis, and he used to only consider mentoring students who had earned an A in an undergraduate complex analysis course. He thought that they needed this foundation to just begin. One semester, his research group consisted of three undergraduates who had been working with him for a semester, when a new, bright-eyed student came to him asking to be in the research group. This student was a sophomore and had not had complex analysis, but he was a hard worker. Michael took a chance and accepted him. He was pleasantly surprised how much he learned from the other undergraduate students in the research group and how much he learned on his own.

Allison runs an REU for students where the minimum requirement is just Calculus III. For projects in fields such as knot theory, combinatorics, and statistics, you don't need every single person in the group to have more background than a few semesters of calculus. If your students are motivated, they can fill in the gaps in their knowledge as they are doing research, possibly with the help of their peer collaborators. What's more, when students learn advanced material because they need it to solve a research problem, rather than simply to pass a course, they learn the material much more deeply.

<u>Step 3: Setting expectations and dealing with group dynamics</u>

Before getting involved in the research, you should discuss some ground rules and expectations with the students. Remember that undergraduates are new to research. There may be norms that you have when working with your peers that undergraduates have not learned or thought about. You probably have a vision of how the project will be organized, but chances are that the students will have a different or hazy vision of how the group should operate. You should sit down with your students and discuss the structure or ground rules for working with you and each other. Some items to discuss are clear, such as when and where you will meet for research. Other items to discuss are: how many hours per week are students expected to work on the project, and what should your students do if they cannot make it to a meeting? Also, it is important to discuss some long-term expectations, such as what end products students will be expected to produce (e.g., a final written report, a presentation to people outside of the research group, a poster).

Another area of discussion that is important but is often not discussed is group dynamics. By group dynamics, we mean how the individual members of the research group interact with each other. Besides differences in mathematical background, students in your research group have differences in ways they communicate with each other and deal with situations. Katherine Leonard [Leo 2008] mentions that some of the group dynamics among the students include lack of communication, sporadic involvement, power struggles, and closure to constructive criticism. In [Cal 2014], Hannah Callender talks about emailing her students before they began researching and asking them to respond to a set of questions such as "In past group experiences, what has worked for you and which hasn't?" "What is the best way for you to receive criticism?" "If you start to struggle, what is your plan?" "What are your current questions or concerns?" During the group's first meeting, Hannah led a conversation about the students' various responses, discussed effective ways to handle conflict and deal with criticism, and talked about each member's different expectations. She wrote, "As one student opens up, it often provides relief for other students who are feeling the same way but are too afraid to admit it." Setting expectations for the project is a critical teaching moment that can help foster an environment where the students are comfortable making contributions and asking for guidance as needed, rather than shutting down when they are challenged.

<u>Step 4: Starting the research and moving it forward</u>

Remember, you have spent a lot of time and effort learning how to do research. Perhaps you've forgotten what it was like when you first started. If you think back to the worries you had in the beginning of your research career, you'll understand that it is an important investment in your students' psyche to discuss the process of doing research with them, especially as they are just beginning. During the research project, it is useful for students to be reminded that it is the rule, rather than the exception, that researchers struggle.

The CURM program [CURM] runs a faculty training 3-day workshop every summer. During the workshop, one of the activities is to create a list of key things that faculty know about research that their students probably don't know. Below is a list of the 12 most common key items. This list is great to share with undergraduate students as they do research. Doing this helps the students learn that feelings of frustration and slow progress are not due to their own lack of mathematical skills, but such feelings are common to all mathematicians doing research. The list includes:

(1) Don't be afraid to ask 'why?'
(2) It's OK if you don't understand an idea the first time (or the second time, or the third time…)
(3) We all get stuck and frustrated. When this happens,
    (a) Take a break.
    (b) Explain to someone (mathematically trained or not) why you are stuck.

(c) Review background material.
(d) See if the problem can be modified (problems are not set in stone).
(e) Check hypotheses or assumptions.
(f) Work out a simple example.
(g) Keep going.

(4) Published work is not always correct—including work of your faculty mentor.
(5) Be open to different ideas and approaches. At some point, you will need an idea or approach that you will have to learn on your own, in which case you will have to think through that idea until you can figure it out yourself.
(6) Your project might go a completely different direction than you think it will.
(7) Everything takes longer than you think it will. Be patient.
(8) It's OK to make mistakes. Making mistakes is a great way to learn!
(9) Hard work and perseverance are necessary (but not sufficient). In fact, hard work is the most important feature of a successful student.
(10) You don't need to know (and cannot know) all the background.
(11) Learn to collaborate.
(12) Research is challenging but rewarding.

In starting to actually do research, it is a good idea to begin by presenting some background material. There are various ways to do this. Some professors prepare notes for students to learn from, some professors have students read material from books or journals that are written at the students' level, and some professors have students read faculty-level research papers. If you choose this last approach, choose the faculty-level research papers judiciously. Once, a colleague came up to Michael and said that he had tried to get an undergraduate student to do research, but he was unsuccessful. Michael's colleague mentioned that he gave a student in his real analysis class his latest research paper in functional analysis. He told the student to read it and drop by when he had questions. The student never came by. Of course, the problem was that the colleague's paper was too advanced for the student to read. It probably would have been difficult for any of us to understand it! So, make sure the material you give to students is accessible to them. Knowing what level material is digestible by students often takes time and experience.

Another mistake that faculty often make in getting students started with research is at the other extreme. Mentors think that they need to explain a tremendous amount of background material before students can begin working on research. This is the paradigm that many of us mentors were exposed to when we began researching in graduate school. Unfortunately, this kind of thinking is one reason some faculty mistakenly believe that undergraduate students cannot do research. These doubtful professors think of undergraduate students doing research in the same way they think of PhD students doing research – the faculty must present a lot of mathematics before the students can start looking at research problems. However, we have found that if undergraduate students focus on just a few essential ideas that they need to start researching, they will pick up many of the other important ideas as they become involved in the work. Moreover, it will be more interesting and enjoyable for them to work on research right away.

Michael once read a colleague's REU proposal in which the colleague wanted to spend the first five or six weeks of the 8-week REU teaching background material to the undergraduate students and then have them work on research during the remaining few weeks. Such an REU proposal would not get funded; the time periods for learning new material and working on research problems were precisely backwards.

We advise you to avoid the common mistake of thinking you have to explain a lot of material before students can begin to start working on research problems. So how much time *is* appropriate for background material? Early on, Michael would present background material during the first 15-25% of the research project time period. During this time, students would work on introductory problems and would learn about potential research problems. Next, he would summarize the potential research problems that they had discussed earlier and let the students choose the problem they wanted to work on. After following this approach for several years, Michael decided to write up notes on the background material and included a few hundred exercises and exploratory problems for the students to do. These notes became two chapters in a book on topics in complex analysis that undergraduate students could explore [BDMRSSS 2012].

Even if you don't intend to write a book, focus on giving students a clear place to start their work. Lara's research is in enumerative combinatorics. She likes to give students a handful of key definitions at the first meeting. The students discuss those definitions until all group members can give appropriate examples to go with them. Then, she gives students two or three possible open questions related to those definitions and encourages the students to explore and express a preference for which question to start with at the second meeting. The students have true ownership of their problem from week 1, rather than much later in the project.

Now, what happens when you start meeting with students who are researching? Our experience is that students appreciate having a structure or a plan for the meetings. When any of us teaches, we prepare what we want to do during the class. When we conduct a committee meeting, we prepare an agenda. When we mentor students, we should also prepare a structured plan. Of course, we can deviate from the plan if a juicy idea comes up that needs some further prodding. We find it is easier to deviate from the plan if we have prepared beforehand.

During the first few meetings with new students, we have them read some material and work on specific problems related to the material. We ask students about the reading and ask them to discuss their solutions to the assigned problems. We encourage the students to work together on the problems. Just as in a class, if one student cannot solve a problem, it's likely that others in the group also need help on it. In that case, we can spend some time discussing that problem as a group. As the students complete the background material phase, we change the structure of the meeting a bit. Now in the meetings, we will review some of the ideas that we have discussed, list specific items that could be worked on, and give an idea of what it would take to work on each item.

For example, suppose the group has been exploring a theorem and, based upon modifying parts of the theorem, we have come up with a conjecture. Then it's good to mention we have several tasks we could explore. First, someone could write up the notes from the day's discussion. Second, someone could do a MathSciNet, arXiv, or internet search to see if another researcher has written a paper related to this conjecture. Third, someone could use a computer to try to generate more examples of this conjecture and see if we can find a counterexample. Fourth, in trying to prove the conjecture, it would be useful to understand the essential ideas behind the proof of the original theorem. So, someone could reread the proof and present to the group the major components of the proof. Fifth, someone could try to prove the conjecture by mimicking the proof of the theorem and inserting the appropriate new components from the conjecture into the parts of the proof where the old components of the theorem are. The students can decide who would like to work on which tasks. Each student picks at least one task, and more than one student may work on the same task. Finally, the students should know that during the next meeting, everyone will be expected to report on the task they have been working on. Typically, most of the tasks have not been completed by the next meeting, but the students should still give a progress report and share any unexpected difficulties. Then, the team will chose new tasks or continue to work on the previous tasks. Students are given some guidance on how to move forward; they are not alone. On the other hand, the students have the freedom to choose an assignment that matches their ability and skills. Also, students have something specific to do before the next meeting.

During the academic year, we usually schedule one or two one-hour meetings a week. The nature of the meeting encourages everyone to talk and share ideas. Students see how research is done in mathematics, they learn problem solving techniques, and they develop independence. Some of these ideas are discussed in [BBDG 2009].

We'd be remiss if we neglected to mention that you should to talk with your students and really get to know them. That's one of the perks of doing this kind of work—building stronger mentoring relationships with students. Occasionally, you can brighten your students' moods and break up the research routine by bringing treats (e.g., donuts or pizza). During summer research programs, you can host game nights, movie nights, or picnics. You can take your students to a baseball game or head to the mountains together for a hike. If you have a sense that your students are working extra hard or are dragging, or if you have something to celebrate, take an impromptu trip to get ice cream!

<u>Step 5: Helping students develop communication skills</u>

Another key difference you'll find in working with undergraduates on research is that they need guidance in writing and presenting mathematics. Developing these communication skills is incredibly important. It requires practice on the students' part and helpful feedback on the professor's part. Michael has talked with recruiters from over 50 companies that hire mathematics students. They have mentioned four things that mathematics students should do besides just majoring in mathematics. Having effective communication skills (i.e., writing and speaking) is one of these top four things. This has changed his thinking about communication skills so that now he requires each undergraduate student to give a talk outside of the mathematics department and the group has to write a research paper. These two activities often have a significant impact on the students. Giving a presentation and writing a paper also brings closure to the project and provides the students with two tangible end products of the research experience that they can put on their résumé or curriculum vitae. Both activities can also be helpful resources for your next research group; when the next group of students makes slides for their presentation, they can use the previous research group's presentation slides as an example of what a talk should look like. The research paper provides some background material written at an appropriate level for your new students to read as they begin their research.

There are many opportunities for students to give a presentation outside of their department. Some colleges/universities have a "celebration" day on campus during which students can present their research. Also, undergraduate students can present at their local MAA sectional meeting, NCUR (National Conferences on Undergraduate Research), the MAA's MathFest meeting, and the Joint Mathematics Meetings. For the latter two meetings, the MAA has a grant that gives some financial support to students who present. You can find details about this online at: http://www.maa.org/programs/students/meetings-conferences/student-travel-grants.

Many students have a mild to intense fear of standing in front of a group of people to talk about their research. The antidote to this fear is practice. The more often students practice giving talks, the more skilled they become at presenting, and the more confidence they develop in their communication abilities. So have students give regular presentations to the group about their research. During an 8-week REU, for instance, you can have students give presentations once a week. After a presentation, the audience members can provide feedback, like: "I did not understand what you meant when you said . . ." or "I think an example or a picture would help people understand what you mean." Outstanding detailed advice on presenting posters and presentations can be found in Higgins et. al. [HLS 2014].

When students write a paper about their research, make sure they start early. One of the most common problems we have heard is that the professor and the students wait too long before beginning to do the writing. As a result, they lose steam and the work never gets written up, or they find themselves in a mad dash to write a paper in a matter of days. To combat this, we have found it helpful to have the students start writing portions of a research report early on (e.g., a first attempt could be done at the midpoint of the project). The students then submit it to us, and we give specific feedback. We ask the students to revise the write-up by incorporating our feedback and include any new material. We recommend that you have your students submit two or three drafts before they get to the last few weeks of the project. Often, the end of a research project is at the end of a semester or at the end of the summer. If the research group waits until the end of the project to start writing the paper, then they will be stressed with all the other obligations at the end of the semester or summer. This will make it hard to complete their task, or at least the final product will be of lower quality. When a research project is over, it is even more difficult to write the paper.

A more proactive approach is to have students write up their work in even smaller pieces than an entire project draft paper. When Lara works with a team of students, she sets up a shared Dropbox folder for the team at the first meeting. Whenever students do computer work, write down a proof, or write up notes from a meeting, the students save their work in this joint repository. In fact, every time a student has an idea for a proof, Lara asks the student to write it up and put it in the Dropbox folder within the week. She returns the draft with comments at the next meeting and takes care of editing on a one-proof-at-a-time basis. This approach has several benefits: writing up one result at a time makes smaller less-intimidating pieces for the students to work with. Also, having a collection of well-polished small documents makes it easier for students to look back at what they've accomplished at any point in the project without losing information. Finally, when the students are ready to write a paper, their focus is on writing the narrative between the proofs since the actual mathematics has been through multiple drafts of editing already!

In any field, writing is a process that benefits from rewriting and revisions. Starting early with the writing process will help to avoid problems later.

<u>Step 6: Preparing for the future</u>

As your project draws to a close, there are some things you can do to help you improve your future self's ability to mentor undergraduate research students. First, have your research students write down research problems they would have liked to work on if they had had more time. Michael and Allison have their research students include this at the end of their research paper. This list of problems can be an excellent source of future problems for new research students. This strategy works especially well if you have your new students read the previous students' research paper as background material. Your new students already have a list of new research problems at the end of the paper they just read.

Second, near the end of the research project, take some time to reflect on your experience. Write down notes on what went well, what didn't go well, and what you would like to do differently next time. This is important to do while the thoughts are fresh in your mind. These notes will help you to improve in your mentoring skills. For instance, at the end of Allison's first summer REU, she discovered that some of her students had struggled with self-doubt and the feeling that they needed to prove themselves earlier in the summer. Knowing this, Allison and the other REU mentors made a note to give students more pep talks and openly address this issue at the beginning of their next REU.

Third, keep a record of your mentoring efforts. This includes keeping a spreadsheet with the names and contact information for the students you worked with, the talks they gave, the papers they wrote, any awards they received, and their future plans. You can also write one or two paragraphs about each student to help you write future letters of recommendation. It is easy to remember all these things right after you work with these students, and maintaining a file with students' information becomes useful as you work with more and more students. Information about your students' accomplishments is beneficial for tenure and promotion, for grant proposals related to undergraduate research, and for teaching awards.

Finally, think of ways you can share the results of your undergraduate research efforts with your dean, institution, and alumni. In doing so, concentrate more on talking about your students and what they accomplished instead of talking about yourself. Take photos of your students working on research and giving talks, so that you can share them with your institution's public relations people to help you promote your work.

Now that we've discussed the Six Fundamental Steps, let's wrap up by summarizing some of the common pitfalls in doing undergraduate research and contrast these pitfalls with productive suggestions. These are:

(1) DON'T give students a research problem that is too difficult or is an all-or-nothing problem. DO pick a problem inspired by other undergraduate work or that you could solve in a short time period. Also, DO be willing to recalibrate as the project evolves.
(2) DON'T wait for difficulties to arise. DO discuss expectations with the students early on.
(3) DON'T try to give too much background material. DO focus on giving ample time to actually work on a research problem.
(4) DON'T improvise your meetings with students. DO provide a predictable structure that encourages everyone to participate and ask questions.
(5) DON'T assume that your students know how to give presentations. DO have the students practice giving talks in a safe space before they give talks in public.
(6) DON'T wait until the end of the research experience to begin writing. DO encourage students to write up drafts of their conjectures and results as they discover them.

We hope this rough guide helps you think more deeply about how to be a superb research mentor, whether you're a novice or a seasoned undergraduate research mentor. If you found what you read here useful, be on the lookout for our upcoming book on mentoring undergraduates in math research.

Acknowledgements

We would like to thank the referees and the guest editors of this edition of PRIMUS for their valuable comments that contributed to the improvement of this paper.

References

[ABJS 2014] F. Alayont, Y. Babenko, C. Jackson and Z. Szaniszlo. "Challenges in promoting undergraduate research in the mathematical sciences," *Involve, a Journal of Mathematics* 7 (2014), no. 3, 265-271.

[BBDG 2009] B. Bailey, M. Budden, M. Dorff, and U. Ghosh-Dastidar. "Undergraduate Research: How Do We Begin?" *MAA Focus*, Jan. 2009, 14-16.

[BDMRSSS 2012] M.A. Brilleslyper, M. J. Dorff, J. M. McDougall, J. S. Rolf, L. E. Schaubroeck, R. L. Stankewitz, K. Stephenson. Explorations in complex analysis. Classroom Resource Materials Series. *Mathematical Association of America, Washington, DC,* 2012.

[Cal 2014] H. L. Callender. "Keys to Successful Mentoring of Undergraduate Research Teams with an Emphasis in Applied Mathematics Research." *Proceedings of the 6th annual International Symposium on Biomathematics and Ecology Education and Research*. Web. 23 April. 2014. http://cas.illinoisstate.edu/ojs/index.php/beer/article/view/796/779

[CUR 2015] The Council on Undergraduate Research. Frequently Asked Questions. http://www.cur.org/about_cur/frequently_asked_questions_/#2

[CURM] The Center for Undergraduate Research in Mathematics. http://curm.byu.edu/.

[Das 2013] K. Das. "From inquiry-based learning to student research in an undergraduate mathematics program." *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies* 23 (2013), no. 9, 829-836.

[Die 2013] J. Dietz. "Creating a culture of inquiry in mathematics program." *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies* 23 (2013), no. 9, 837-859.

[Dor 2014] M. Dorff . "Nonacademic careers, internships, and undergraduate research." *Involve, a Journal of Mathematics* 7 (2014), no. 3, 303-313.

[DH 2003] M. Dorff and L. Hall. "Solids in $\mathbb{R}^n$ whose area is the derivative of the volume." *The College Math. J*. 34 (2003), no. 5, 350-358.

[EMP 2013] J. Ellis-Monaghan and G. Pangborn. "An example of practical organization for undergraduate research experiences." *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies* 23 (2013), no. 9, 805-814.

[EGK 2015] P. Etingof, S. Gerovitch, and T. Khovanova. "Mathematical Research in High School: The PRIMES Experience." *AMS Notices* 62 (2015), no. 8, 910-918.

[GS2007] R. Gillman and Z. Szaniszlo. "Academic Year Research at Valparaiso University." *Proceedings of the Conference on Promoting Undergraduate Research in Mathematics*. American Mathematical Society, (2007), 279-283.

[HLS 2014] A. Higgins, L. Ludwig and B. Servatius. "Papers, posters, and presentations as outlets for undergraduate research." *Involve, a Journal of Mathematics* 7 (2014), no. 3, 327-333.

[Leo 2008] K. Leonard. "Adventures in Academic Year Undergraduate Research." *AMS Notices* 55 (2008), no. 11, 1422-1426.

[MV 2014] C. McBee and V. Vasilevska. "Information for faculty new to undergraduate research." *Involve, a Journal of Mathematics* 7 (2014), no. 3, 395-401.

[Rob 2013] G. Roberts. "Conducting Mathematical Research with Undergraduates." *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies* 23 (2013), no. 9, 785-797.