REVIEWS

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An Introduction to Undergraduate Research in Computational and Mathematical Biology. Edited by Hannah Callender Highlander, Alex Capaldi, & Carrie Diaz Eaton. Birkhäuser, Basel 2020. 469 + xii pp., ISBN 978-3-030-33644-8, $70.

A Project-Based Guide to Undergraduate Research in Mathematics. Edited by Pamela Harris, Erik Insko, & Aaron Wootton. Birkhäuser, Basel 2020. 324 + xi pp., ISBN 978-3-030-37852-3, $70.

Reviewed by **Lara Pudwell**

Which comes first: the undergraduate research problem or the mentoring style? There are a variety of ways to answer this question. A problem-first strategy may see the mathematics as primary. In other words, if one can pick a great problem, then various mentoring tactics can be employed to help students work through different phases towards a solution. The mathematics provides the outline and the mentor's function is to coach the student through learning definitions and background, scaling up to working on a new problem; in some sense, the student is an apprentice learning alongside the faculty researcher. On the other hand, a faculty member may have a particular mentoring process, where the mathematics is secondary, but different steps of student professional development provide the outline for the experience. The student still learns the phases of mathematics research from searching the literature to asking and refining a research question to building a solution, but the publishability of the result is less important than the student building a new skill set. In this situation the faculty/student relationship may have less of an apprentice model; the faculty member is more of a knowledgeable resource.

Of course, we could ask if it's necessary to make either the problem or the mentoring style primary; some undergraduate research experiences don't fit cleanly into one model or the other, and are constantly adapting along a spectrum of possibilities. But both of these factors are necessarily in play in any undergraduate research experience: mentoring without a piece of mathematics isn't developing new mathematical knowledge, while mathematics without a mentoring style misses out on learning professional tools from other practitioners. And faculty new to mentoring undergraduate research projects need strategies both for finding problems and for providing direction to students.

No matter your response to this chicken-egg type dilemma, there are certainly qualities of research problems that contribute to a greater chance of a successful undergraduate research experience. In [**?**], Michael Dorff, Allison Henrich, and I argue that the hallmarks of an excellent undergraduate research problem are:

1. The problem should require a limited amount of background material.
2. The problem should be specific and concrete.
3. The problem should have multiple layers, starting simple and becoming more difficult.
4. The problem should be of interest to you and your students.
5. The problem should lend itself to studying examples and using computers.
6. The problem is one that the faculty mentor has some idea how to solve.

This last characteristic can be interpreted in multiple ways. Many faculty have maintained successful research agendas over the years, and then adapted threads of their research to invite the participation of undergraduate students. They're not necessarily looking for new topics, but for new spinoffs within their existing work. But this is not the only approach. It's certainly possible to have some idea where to find tools for approaching a new problem without the research topic coming from your own area of expertise. In this latter scenario, the faculty member may have an added challenge of not knowing the existing literature well enough to confidently locate questions that are actually new.

Rather than reinventing the wheel, the *Foundations for Undergraduate Research in Mathematics* (FURM) series is designed to help faculty and students in this situation, presenting collections of chapters written by seasoned faculty mentors. Each chapter contains mathematical background for faculty and/or students to familiarize themselves with a new area (Characteristic 1: limited amount of background material). Each chapter contains a variety of exercises that scaffold up to research project suggestions (Characteristic 3: multiple layers). And each chapter ends with an extensive bibliography to help ground the project ideas in existing literature (Characteristic 4, extended: these topics are also of interest to the broader mathematics community). A skeptical reader might wonder as to the purpose of these books, especially as gathering specific and concrete problem ideas from a book may sound like a recipe for your students getting scooped while other faculty mentors collect the same specific, concrete problem ideas and try them out simultaneously with their own students. Let me allay these concerns by discussing the specific content of the two books under review.

In full disclosure, I'm reading these books as a pure mathematician. What is striking to me, then, is that the more applied volume (*An Introduction to Undergraduate Research in Computational and Mathematical Biology*) provided the more direct response to this first critique: while many faculty could read the same chapter for inspiration, this volume is designed in such a way that getting scooped seems unlikely. Already on page 2 of this book, P.J. Hurtado comments:

> A great recipe for an undergraduate research project is to (a) find an interesting paper in a reputable, peer-reviewed journal that includes some analysis of an ODE model, (b) repeat one or more analyses to confirm a specific published result, and then (c) answer similar questions via the analysis of a new model derived by modifying one or more assumptions of that original ODE model.

Later, in a thorough chapter on experimentation via numerical methods, Brittany E. Bannish and Sean M. Laverty suggest,

> To start a mathematical modeling research project, you can find a published model and modify it somehow, or learn the relevant biology needed to comfortably write your own model... Once the model has been written, you can quickly solve it numerically to see if results make sense. Once you obtain reasonable results, you can start to experiment in earnest, or perhaps reach out to experts in the field.

Multiple authors make the point that starting with an existing model and modifying it can help improve the model and determine which assumptions and parameters have the biggest impact on the model's results.

Even when they learn the same modeling tools, students can generate a myriad of real-world topics where those tools are relevant. Drawing from decades of experience mentoring students at the Mathematical and Theoretical Biology Institute (MTBI) at Arizona State University, Carlos W. Castillo-Garsow and Carlos Castillo-Chavez write of a model where

> ... students form self-selected groups of three to five undergraduates, and investigate a problem of their own choosing. They research the background of the problem, identify a question, construct a model to address the question, analyze the results, and write a technical report describing their project...Students take the lead on the project and provide subject matter knowledge, while mentors provide general expertise in mathematical modeling techniques that can be applied to a broad variety of topics.

They elaborate on how discussing the fundamental tools of modeling with differential equations or of agent based modeling, beyond what a typical undergraduate may see in their standard course work opens up options for a variety of creative ways to generate problems that match student interest, and without fear of reinventing a previously-studied project.

The writing in a compilation book is almost by definition uneven. When each chapter has different authors, not only are different writing styles at work, but so are different mentoring and mathematics philosophies. At least three different chapter authors flesh out the compartmental SIR model, or a variation thereof. If one were reading the entire text in sequence, perhaps such pervasive models could be discussed once in an introductory chapter, so that authors can more quickly get to their particular focus. But that doesn't seem to be the intended reading model here. Instead, it may be more useful to think of a FURM volume as a collection of short stories; you'll only need to select one story at a time to work through with students.

Most chapters alternate between discussion and exercises that serve as an organized way to coach students through new definitions and tools. For many chapters, I could envision assigning a few pages a week for an undergraduate to work through, including completing exercises, as a way to scaffold up enough new mathematical skills to put them into action on a project. Ultimately, each chapter author suggests a few research projects, which could be particular problems or could be quite general, with room for students and faculty mentors to determine their own focus. As a warning, each chapter begins with a list of suggested prerequisites, and those should be taken seriously! No mathematician is an expert in everything, and there was a high correlation between how many details I got out of each chapter and how many of the prerequisites I had meaningful prior experience with. Read as an entire collection, rather than one chapter at a time, I came away with a better appreciation for the range of topics undergraduates can delve into in biomathematics. I canonically expected to see models for infectious disease, population growth, and predator-prey interaction. But there was a wealth of other topics where the mathematics was new to me, including antibiotic resistance via agent-based modeling, identifying bird songs via neural networks, and studying fluid flow in the lungs using tools of multivariable calculus. Agent-based modeling and differential equations are pervasive throughout the text, but a variety of other mathematics comes into play, especially in later chapters, such as random graphs and hidden markov models. Many of the chapters really shine with detailed examples and computer code in R and Netlogo to help readers learn computational skills at the same time as learning new material. I was truly convinced at the end of this particular volume that if I had an interest to switch to applied mathematics with an eye towards biological applications, that this text would be a handy reference to get started once I settled on a particular chapter to delve into.

So how does a more pure mathematics book take on this same challenge? *A Project-Based Guide to Undergraduate Research in Mathematics* is certainly not only a pure mathematics book. For example, the chapter by Elizabeth Drellich and Heather C. Smith takes a problem inspired by RNA folding and presents it combinatorially. While one could focus on the mathematics in isolation, they make the case that an infinite number of variations exist either by varying the mathematical constraints of the problem or by looking back to RNA for new biological constraints. Similarly, Elizabeth Gross, Colby Long, and Joseph Rusinko motivate their work by discussing evolutionary links between species, and Pamela E. Harris, Erik Insko, and Katie Johnson discuss a combinatorial problem motivated by broadcast signals. While each of these chapters really could be read only for the pure mathematics, the motivating applications open up a wide array of variations to consider.

Even the chapters that are presented more as pure mathematics from the beginning hint at interesting interplays – between tropical mathematics and scheduling problems, between chip-firing games and group theory, between the variety of ways to define trigonometric functions and generalize them – and these multiple perspectives invite a range of follow up projects, rather than one linear path of what readers ought to consider next. Still other chapters lay a firm foundation in terms of definitions and known results, but quickly branch out into uncharted territory. The chapter on tiling questions by Steve Butler, Jason Ekstrand, and Steven Osborne was particularly charming, not only for their entertaining and informative footnote commentary, but because it built up relevant tools in a natural way and then opened out into a wide variety of general ideas like "almost nothing is known about this problem generalized to three dimensions". They also give a clear description of what would be a deep enough result to merit publication in their topic area.

A strength of the *Project-Based Guide* is its range. The projects draw from algebra, combinatorics, geometry, analysis and more. The book also contains a chapter on RUME (Research in Undergraduate Mathematics Education). Instead of a new researcher guessing what research looks like in a more qualitative setting, Milos Savic gives a variety of ideas on how to start an education project and clearly lays out the expected sections for a RUME publication.

Overall, whether looking for a new project in applied or pure mathematics, both books handedly counter the concern that getting research ideas from others means one is likely to be scooped. These project suggestions at their best have such a range of variation that interested readers will almost certainly find their own take that is indeed new.

For many faculty members working on research with undergraduate students, the final mathematical results of an undergraduate research experience may be secondary to developing professional skills like communication, interaction with the literature, and more. These faculty may be looking for help understanding strategies for getting started with new students more than they are looking for the mathematical content. To this end, some chapter authors in each volume include explicit mentoring tips that grow out of their own experience. The format of this advice varies from author to author. For example, Carlos W. Castillo-Garsow and Carlos Castillo-Chavez include an explicit "Notes for Mentors" section about their process at MTBI. Stephan Ramon Garcia gives 21 principles for mentoring undergraduate research at the outset of his chapter of *A Project-Based Guide...*. Both of these chapters include sequences of previous student projects where the authors give a clear story of how the specific project topics were originally generated. Also in the project-based guide, Pamela E. Harris, Erik Insko, and Katie Johnson devote an entire section to "Developing Accessible Research Projects". Alicia Prieto-Langarica co-authors a chapter in each volume, and both of these chapters end with a section written by her research students; in the biology-focused text two students each write personal reflections on their professional growth through the research experience, while in the project-focused text, one student writes up the mathematics of their project. In both cases, it's refreshing to see not just what the faculty member thinks is going on, but to hear from students in their own words. Although one can implicitly conjecture the mentoring strategy that leads from a sequence of exercises to a list of project suggestions, it's helpful to hear authors (and their students!) distill what they've learned about the mentoring process more directly.

The two books discussed here are the second and third installments in the FURM series, with more volumes in the works. Although the writing and style understandably vary from chapter to chapter, if one takes the prerequisites seriously, working through a chapter with an undergraduate should provide an excellent on-ramp to a new and interesting research project. For those looking for more explicit mentoring tips, the quantity of explicit advice in the FURM volume depends on the chapter authors, but there are other tools that one can turn to for this. In [**?**], the focus is largely on logistics of mentoring research projects in mathematics rather than project material. The Council on Undergraduate Research [**?**] provides a wealth of resources about best practices for undergraduate research across disciplines. The Undergraduate Research Special Interest Group of the MAA (UR SIGMAA) provides regular opportunities at national meetings to network with other mentors and learn from them. Together, the Mathematics, Computer Sciences, and Statistics division of CUR and UR SIGMAA released a document of best practices for mentoring undergraduate research in a virtual environment, which is freely available from the CUR division's webpage [**?**]. Nonetheless, the FURM series is a valuable contribution from a different angle than these professional societies. If you want a glimpse into other researchers' project development process, these texts provide many such examples, and you can pick and choose from the ones that most directly match your background and interests. The next step? Pick a chapter and dive in!

References

[1] The Council on Undergraduate Research. `https://www.cur.org/`.

[2] Dorff, M., A. Henrich, L. Pudwell (2019). *A Mathematician's Practical Guide to Mentoring Undergraduate Research*,Washington, DC: Mathematical Association of America.

[3] Mathematics, Computer Sciences, and Statistics Division of CUR, UR SIGMAA, `http://www.mathcscur.org/index.php/aboutmathcscur/ur-sigmaa/`.

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