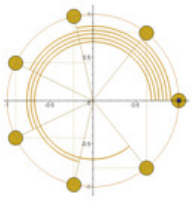

Roots of Unity

Mathematics: learning it, doing it, celebrating it.

Roots of Unity Home

Mathematicians Predict What's in Your Wallet

By Evelyn Lamb | June 20, 2013 | 9

It's all about the Washingtons. And that smiley face button. Image: flickr user xJason.Rogersx

When I go to Europe, my pockets rapidly fill up with change. In addition to language barriers that prevent me from quickly understanding how much I owe, I have trouble dealing with the unfamiliar coin denominations. The best way to make 75 cents is to use a fifty cent piece, one twenty, and a five, not three quarters.* But I have trouble remembering that on the fly. The one- and two-euro coins further confuse the issue for me, as I reach for a bill rather than coins for a three euro transaction. In the airport on the way home, I generally try to convert as many of my amassed coins to duty-free chocolate as possible, but I often come home with quite a few coins jingling in my pocket anyway.

In the states, I try to avoid using coins in my everyday life. I rarely carry a purse, and my wallet doesn't have a coin pocket, so they just clang around in my pockets. I let my non-quarters accumulate in a repurposed coffee can, and every few years I count them up and treat myself to a sandwich or something. The quarters are immediately diverted to the laundry fund.

In a paper posted to the preprint site arXiv.org on June 9, mathematicians Lara Pudwell of Valparaiso University and Eric Rowland of the University of Quebec at Montreal tackled the question of how many coins people have sitting around. They used statistical techniques to determine that a typical carrier of US currency is most likely carrying 10 coins at any given time: 1 quarter, 1 dime, 1 nickel, and 7 pennies. If,

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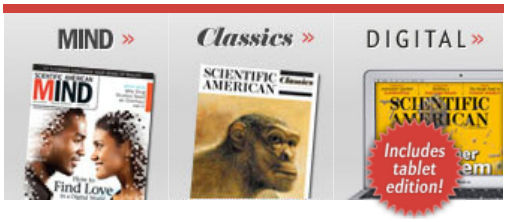

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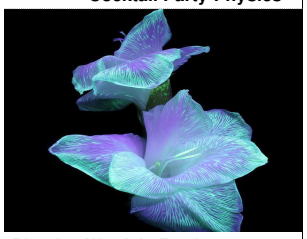

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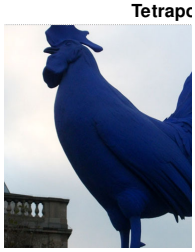

Right now, there's a g chicken in Trafalgar S

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Observations

like me, you save your coins instead of spending them, then it's likely that about 31.9 percent of your piggybank contents are quarters, 17 percent are dimes, 8.5 percent are nickels, and 42.6 percent are pennies.

But the assumptions and methods that go into the analysis are more interesting than the conclusions by themselves. It's a pretty fun and readable paper, so I'd encourage you to check it out for yourself. But if you want the Cliffs Notes, read on.

With any mathematical model of real-world behavior, researchers have to decide what assumptions to make. Pudwell and Rowland start the paper with two reasonable assumptions about the everyday use of coins:

"(1) The fractional parts of prices are distributed uniformly between 0 and 99 cents. (2) Cashiers return change using the fewest possible coins."

Incidentally, I'm curious about the first assumption. I'd guess that if people always bought items one at a time, prices of their transactions would not be uniformly distributed because a lot of prices end in .99 or .49, and depending on how taxes work in your city, that probably makes certain prices more likely than others. But I'd imagine that for multi-item transactions such as a trip to the grocery store, a uniform distribution of the cents part of prices is pretty likely. It would be interesting to see whether the tax rates and average prices of goods in certain locations bias the prices in favor of certain cents amounts, but this is a matter of going out and collecting a lot of data, not making models and feeding them to computers.

For most of the examples in the paper, the researchers also assume the use of US currency denominations (1-cent penny, 5-cent nickel, 10-cent dime, and 25-cent quarter—half dollars and dollar coins are not included), but their model can handle any set of coin denominations.

Pudwell and Rowland divide the world into two groups depending on coin handling preferences: those who pay for everything with bills and keep coins in a jar at home, and those who do use coins when they pay for things. The situation of the former, the "coin keeper," is relatively easy to handle. To figure out the proportions of different coins in a coin keeper's stash, you just have to tally up how many coins you get for every possible price. If the cents amount of prices are truly equally distributed, this will give you the average percentage over a large number of transactions.

The situation for the coin spender is a bit more complicated. With the additional assumption that a coin spender keeps less than $1.00 in coins in her wallet at all times, Pudwell and Rowland note that a spender's choices can be modeled as a Markov chain, meaning that there are a finite number of possible coin combinations in a wallet (called "wallet states"), and there is an unambiguous process that happens when the spender buys something. For each transaction, the new wallet state only depends on the wallet state directly before it, the amount paid at the store, and an algorithm for how a spender pays for things.

For example, the "big spender" overpays as little as possible and uses as few coins as possible if there are multiple ways to overpay as little as possible. If a big spender has to pay 27 cents, she will use a quarter and a nickel over three dimes if she can't make exact change. Overpaying as little as possible takes precedence over minimizing the number of coins: if she had only a quarter and three dimes, she would spend the three dimes rather than the quarter and one dime. (In this case, the choice to overpay

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as little as possible rewards her by decreasing the number of coins she receives as change.)

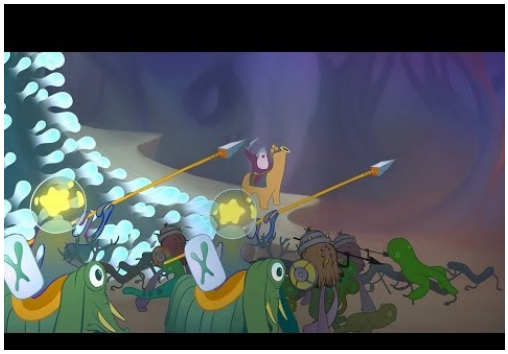
This is the part of the post where it's helpful to know a little bit of linear algebra. If you don't feel like thinking about linear algebra right now, skip the rest of this paragraph. The method that Pudwell and Rowland used to compute the probabilities of different "wallet states" involves making a matrix that keeps track of how likely it is to move from one wallet state to another during one transaction. A particular eigenvector of one of this matrix tells you the likelihood of each wallet state after a long period of time. The only catch is that for real-world currencies and prices, the matrices are ginormous, to use a technical term. There are 6,720 different combinations of coins that add up to under $1.00 in the US currency system, so you end up with a 6720×6720 matrix in the case of the "big spender." Just running the computations necessary to create the 6720×6720 matrix took 1 day's worth of CPU time. Pudwell and Rowland use numerical approximation to find the eigenvectors rather than computing them exactly.

My favorite thing about this paper is the thought that went into the different examples Pudwell and Rowland discuss. They analyze several types of coin spending habits based on how people actually spend their cash. In addition to the "big spender" described above, they analyze the "pennyless" (not to be confused with "penniless") purchaser, who leaves pennies in those "give a penny/take a penny" trays rather than giving or receiving them as change, and the "quarter hoarder." In their words, "the quarter hoarder is a spending strategy utilized by college students and apartment dwellers who save all their quarters for laundry. All quarters they receive as change are immediately thrown into their laundry fund, so the quarter hoarder's wallet contains only dimes, nickels, and pennies." It's like they're reading my mind! They also analyze what would happen with a fictional currency that replaces the dime with an 18-cent piece. They chose this particular fictional currency because it is the currency system that minimizes the average number of coins given as change. For each variation, they computed the most likely combinations of coins for people using that system.

At the end of the article, Pudwell and Rowland speculate about some of their assumptions and wonder how well they match the way people actually behave. They assume that people want to minimize the number of coins they spend, but perhaps it makes more sense to maximize the number of coins in a transaction, thereby minimizing the weight in your pocket after the transaction. "Of course, the best way to minimize the number of coins in your wallet is to curtly throw all your coins at the cashier and make them give you change. But...we are strictly interested in *civil* models of spending." They suggest some interesting ways to modify the algorithm to take different spending patterns into account.

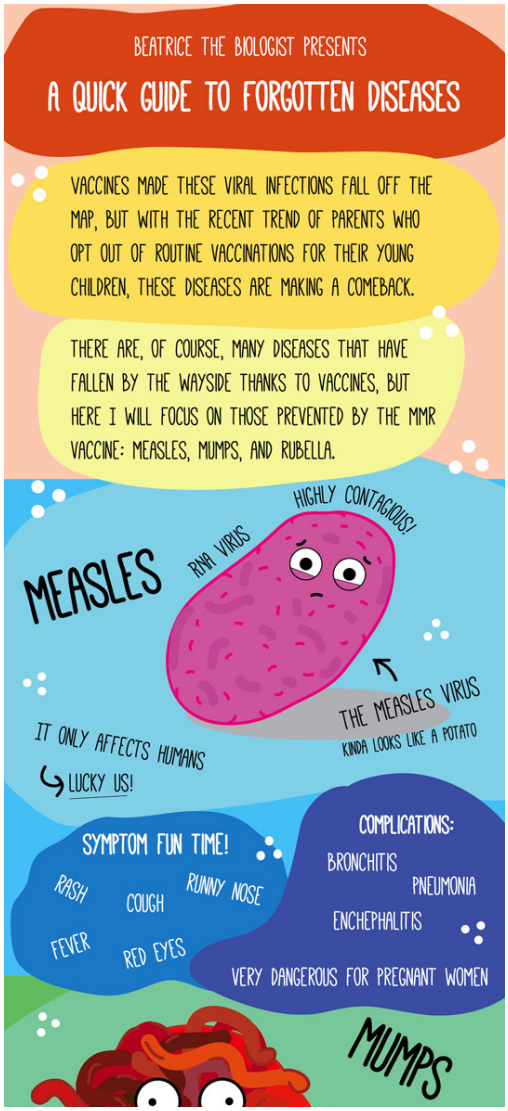
I decided to dump out my coin jar to see how closely my coin hoarding habits align with what the model predicts. Because I am a hybrid of the coin keeper and the quarter hoarder, I only considered non-quarters. I have 80 dimes (20.3 percent of my total change), 27 nickels (6.8 percent), and 288 pennies (72.9 percent). Ignoring the quarters in the "coin keeper" model, Pudwell and Rowland predict 25 percent dimes, 12.5 percent nickels, and 62.5 percent pennies. So my numbers aren't too far from the predicted amounts. As disappointing as it is to be so typical, it's kind of cool that a mathematical model that doesn't even know me did a pretty good job of guessing what's in my coin stash.

Video of the Week



Good Microbes make Good Pets

Image of the Week



Vaccinate! Do it for the testicles

What are your coin carrying habits? Are you a hybrid of some of the coin types discussed above, or do you use a different algorithm for your cash transactions? Please share in the comments. For more details about what Pudwell and Rowland think your wallet holds, check out the paper on the arXiv.

*This sentence was changed after publication to correct an arithmetic error.

About the Author: Evelyn Lamb is a postdoc at the University of Utah. She writes about mathematics and other cool stuff. Follow on Twitter @evelynjlamb.
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9 Comments

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1. chrisirwindavis
12:38 pm 06/20/2013

"The best way to make 75 cents is to use a fifty cent piece, two twenties, and a five, not three quarters."

50 + (2×20) + 5 = 95, not 75
Perhaps you meant *one* twenty?

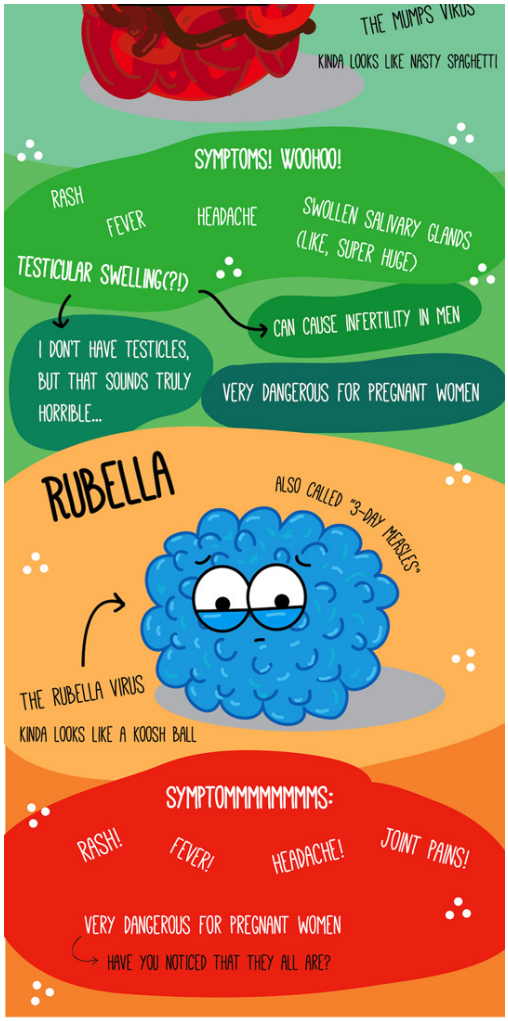
Normally I wouldn't comment on a trivial typo, but this IS a math article.

Link to this

2. OgreMk5
1:29 pm 06/20/2013

Totally dump all coins in a jar. Since I rarely have more than one cash transaction per day (more like per week), I rarely have coins. Even my snack machine purchases are via a refillable card on my work badge.

Link to this



3. marclevesque
2:21 pm 06/20/2013

Most of the time when I pay for something I give the maximum number of coins. So if I made a 27 cent purchase I'd give 3 dimes instead of a quarter and a nickel.

But I'm Canadian and the penny was just abolished, prices are still advertised with pennies but the total at the cash is rounded, so for a 27 cent total I pay 25 cents and for a 28 cent total I pay 30 cents.

Link to this

4. dadster
4:02 pm 06/20/2013

Very interesting . When can we see a mathematical model of a mind . Does statistics apply to the analysis of the mind ? Say can statistical prediction be made as to the frequency of sadness and happiness and the average time those states last in a normal healthy mind ?

Link to this

5. Evelyn Lamb
5:56 pm 06/20/2013

Yep, I made a typo. If you spend much time with mathematicians, you will find to your dismay that we make about as many arithmetic errors as everyone else. But I definitely should have caught it!

Link to this

6. wakeele
3:38 pm 06/21/2013

If one has such trouble with using Euros or any foreign currency over US currency, then that person probably shouldn't be writing an article on math, much less anything for Scientific American.

Link to this

7. Wayne Williamson
3:57 pm 06/21/2013

Interesting, very rarely use cash any more and if the change is less than 50 cents, I either donate it to one of the many boxes sitting on the counter or put it in the "free" tray for someone else to use.

Link to this

8. wgboulet
11:28 am 06/27/2013

I'm the kind of guy who keeps all his change in the car and scoops it up before going to make any cash purchase. Then I pay with as many coins as I can get away with, and am always pleased when I come back empty-handed (which doesn't happen very often). Apart from the predicting how many coins you have in your pocket at any given time, it would be interesting to have an evolutionary psychologist tell us why some people "hoard" and others "splurge".

Being a Canadian, there are no more pennies in my collection and Eric Rowland, who works in Montréal, may have to rework his model to account for that situation.

Link to this

9. evelynjlamb
3:49 pm 06/27/2013

The paper does contain what I assume is a subtle dig at Canada's recent pennylessness: "Let's require that the currency contains a 1-cent coin, since a reasonable currency system has a denomination for its unit."

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